點型態分析：最鄰近分析法
Point Pattern Analysis: Nearest Neighbor Analysis

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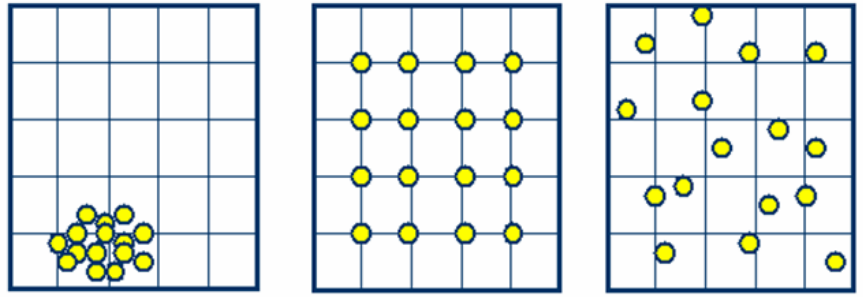
Outline

- Analyzing global point patterns: Overview
- Nearest Neighbor Analysis (NNA)
  - Test statistic: R scale
  - Statistical Significance Test
    - Comparing with **theoretical random pattern**
    - Monte Carlo significance test
  - K-order NNA
- G(d) Function

Point Pattern Analysis

- Analyzing *Global* Patterns
  - Nearest Neighbor Analysis
  - *Distance*-based Functions
  - *Density*-based Methods



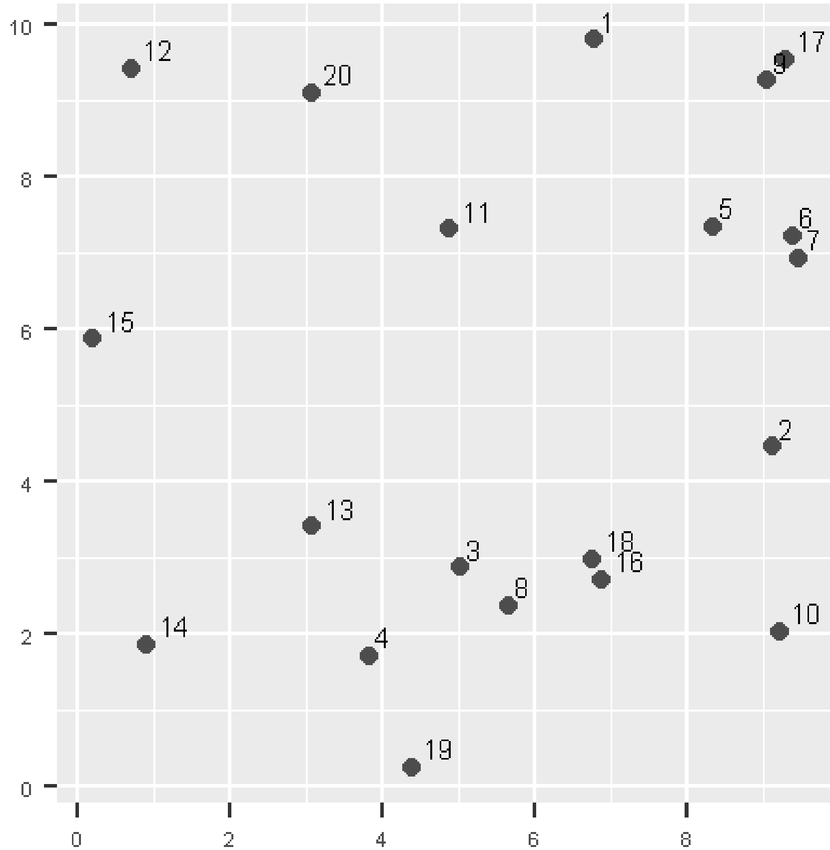
(a) Clustering (b) Dispersion/Uniform (c) Random

Properties of Spatial Point Patterns

- First-order Property (measuring the **intensity**)
  - **First order property** indicates the intensity of a process, mean number of events per unit area at point p.
  - The concept is similar to the **mean** as the first order statistics in statistical theory.
- Second-order Property (measuring the **dependency**)
  - **Second order property** of a spatial point pattern indicates spatial dependency of a process, mean number of paired events per unit area, like between point p and q.

Nearest Neighbor Analysis (NNA)

measures the average distance from each point in the study area to **its nearest point**.



From	To	Distance	From	To	Distance
1	9	2.32	11	20	2.55
2	10	2.43	12	20	2.39
3	8	0.81	13	4	1.85
4	19	1.56	14	13	2.67
5	6	1.05	15	12	3.58
6	7	0.3	16	18	0.29
7	6	0.3	17	9	0.37
8	3	0.81	18	16	0.29
9	17	0.37	19	4	1.56
10	2	2.43	20	12	2.39

Nearest Neighbor Analysis (NNA)

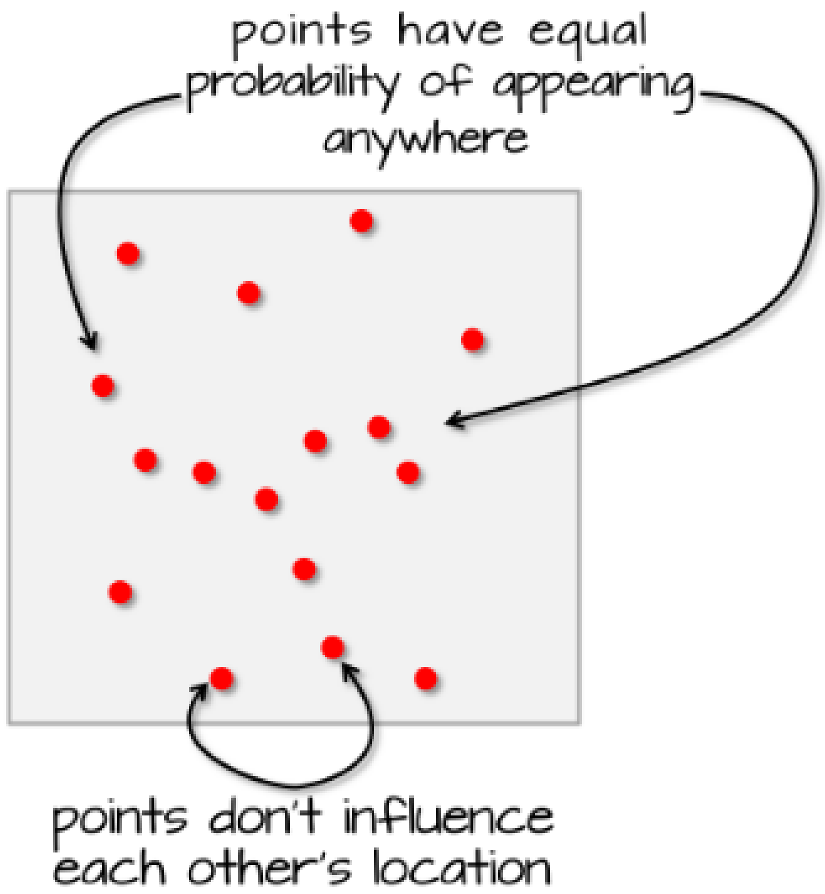
- The NNA compares the average distance between nearest neighbors to that of a random pattern
  - 大於 隨機分布的平均距離 : Dispersed
  - 小於 隨機分布的平均距離 : Clustered

- R Scale（r_{obs} / r_{exp}）
  - For theoretical random pattern: $r_{exp} = \dfrac{0.5}{\sqrt{n / A}}$

Theoretical Random Pattern

- Spatial analysis techniques compare observed point patterns to ones generated by an **independent random process** (IRP) also called **complete spatial randomness** (CSR). CSR/IRP satisfy two conditions:
- Any event has equal probability of being in any location, a **1st order effect**.
- The location of one event is independent of the location of another event, a **2nd order effect**.

Theoretical Random Pattern



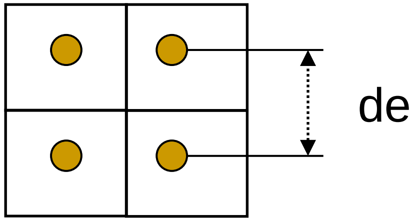
Average nearest distance of theoretical random pattern

Dispersed/ uniform

The expected mean distance for a completely dispersed distribution

$$\bar{d}_e = \frac{1}{\sqrt{n / A}}$$

Divide the number of features by the area of the study area, and take the square root...then divide the result into 1



Random

The expected mean distance for a random distribution

$$\bar{d}_e = \frac{0.5}{\sqrt{n / A}}$$

Divide 0.5 by the square root of the number of features divided by the area

Point	Nearest Neighbor	Distance
1	2	1
2	3	0.1
3	2	0.1
4	5	1
5	4	1
6	5	2
7	6	2.7
8	10	1
9	10	1
10	9	1
		10.9

r	1.09
Area of Region	50
Density	0.2
Expected Mean	1.118034
R	0.974926

Point	Nearest Neighbor	Distance
1	3	2.2
2	4	2.2
3	4	2.2
4	5	2.2
5	7	2.2
6	7	2.2
7	8	2.2
8	9	2.2
9	10	2.2
10	9	2.2
		22

r	2.2
Area of Region	50
Density	0.2
Expected Mean	1.118034
R	1.96774

Point	Nearest Neighbor	Distance
1	2	0.1
2	3	0.1
3	2	0.1
4	5	0.1
5	4	0.1
6	5	0.1
7	6	0.1
8	9	0.1
9	10	0.1
10	9	0.1
		1

r	0.1
Area of Region	50
Density	0.2
Expected Mean	1.118034
R	0.089443

$$\bar{r} = \frac{\sum r}{n}$$

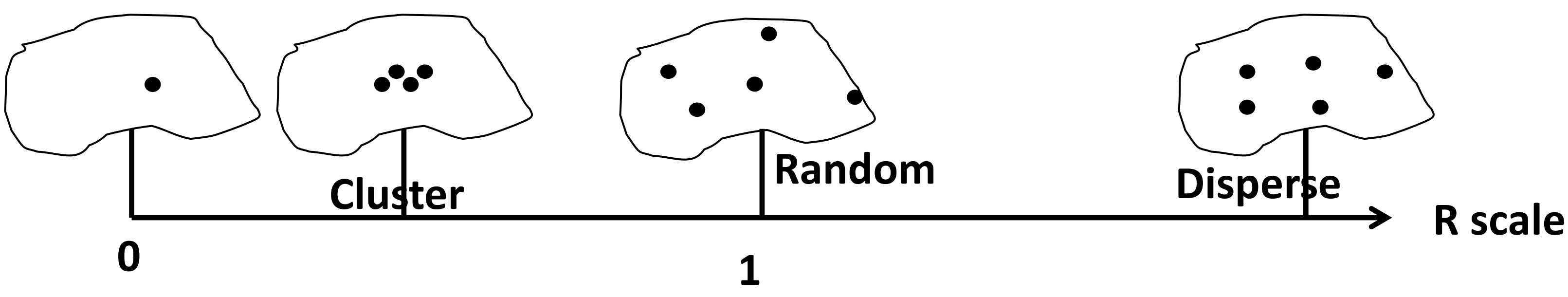
$$d = \frac{n}{area}$$

$$\bar{r}(e) = \frac{.5}{\sqrt{d}}$$

$$R = \frac{r}{\bar{r}(e)}$$

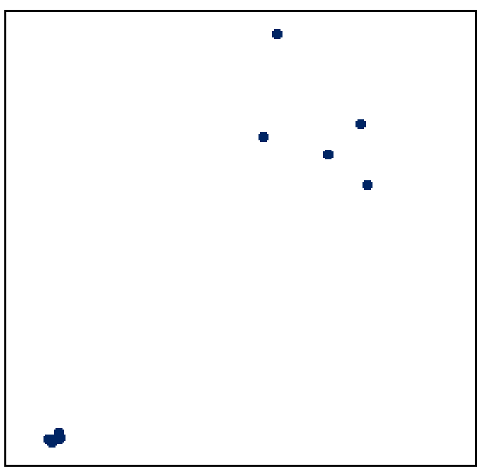
R Scale

- R = 0 (completely clustered)
- R = 1 (random)
- R = 2.149 (completely dispersed)



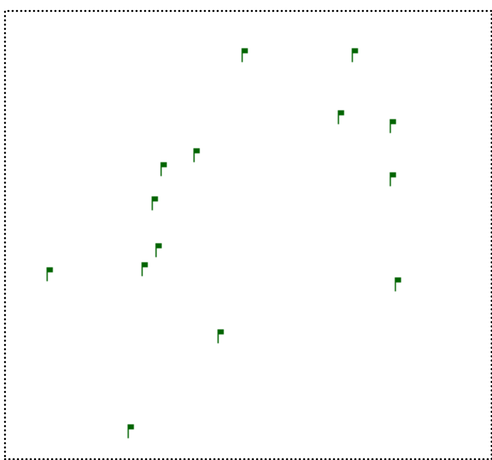
Average Nearest Neighbor Distance

	D.obs	D.exp	R	Result
情境 1	3.157	7.958	0.397	Cluster
情境 2	40	13.5	2.424	Disperse
情境 3	14.486	11.207	1.293	Random



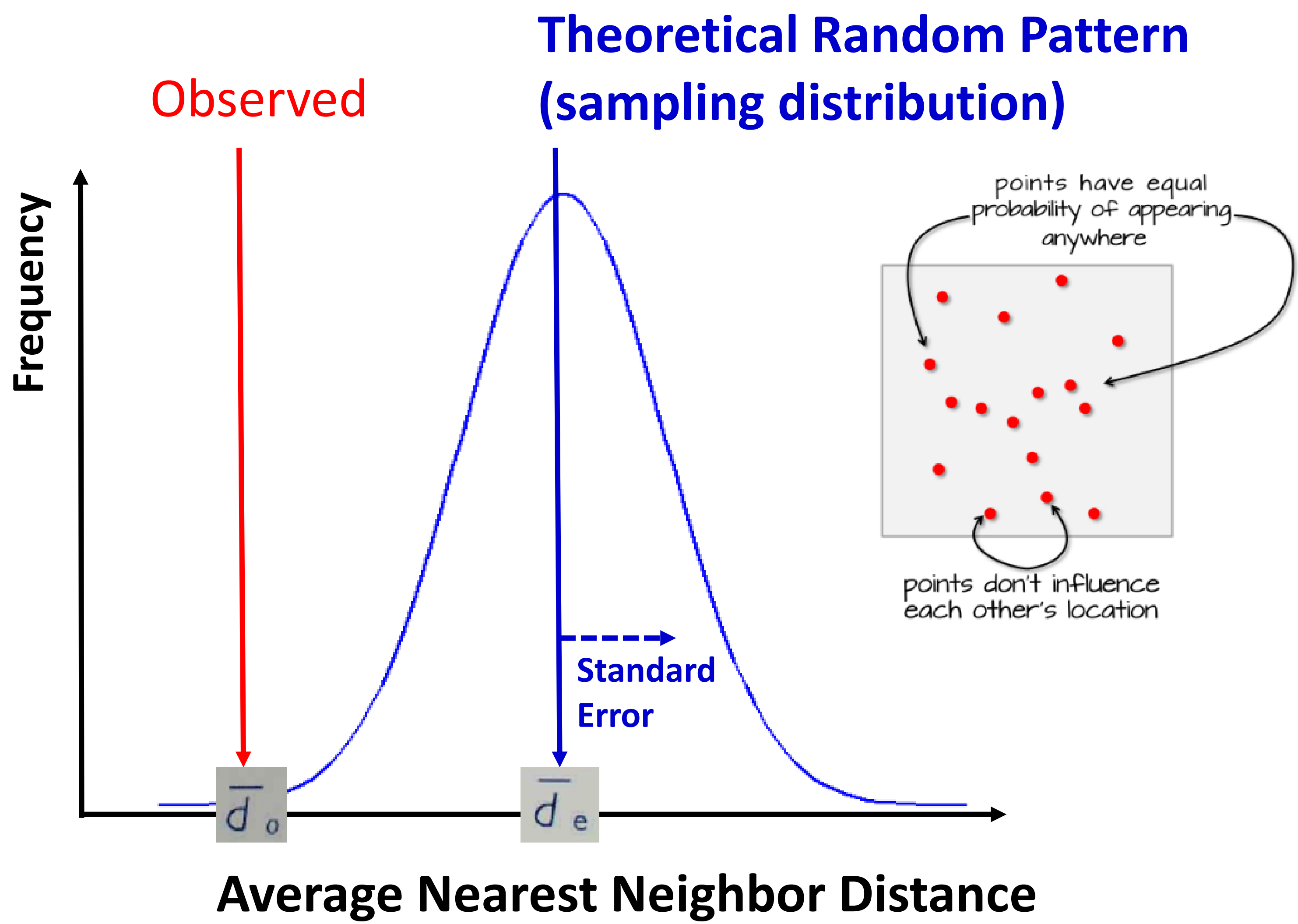
情境1

情境2



情境3

Significance Test of NNA:
1. Comparing with Theoretical Random Pattern



Significance Test of NNA

The expected mean distance is subtracted from the observed mean distance....

The Z-score

$$Z = \frac{\bar{d}_o - \bar{d}_e}{SE}$$

....and the difference divided by the standard error

The expected mean distance for a random distribution

$$\bar{d}_e = \frac{0.5}{\sqrt{n / A}}$$

The standard error

0.26136 (a mathematical constant) is divided by....

$$SE = \frac{0.26136}{\sqrt{n^2 / A}}$$

....the square root of the number of features squared (n^2) divided by the areal extent (A) of the study area

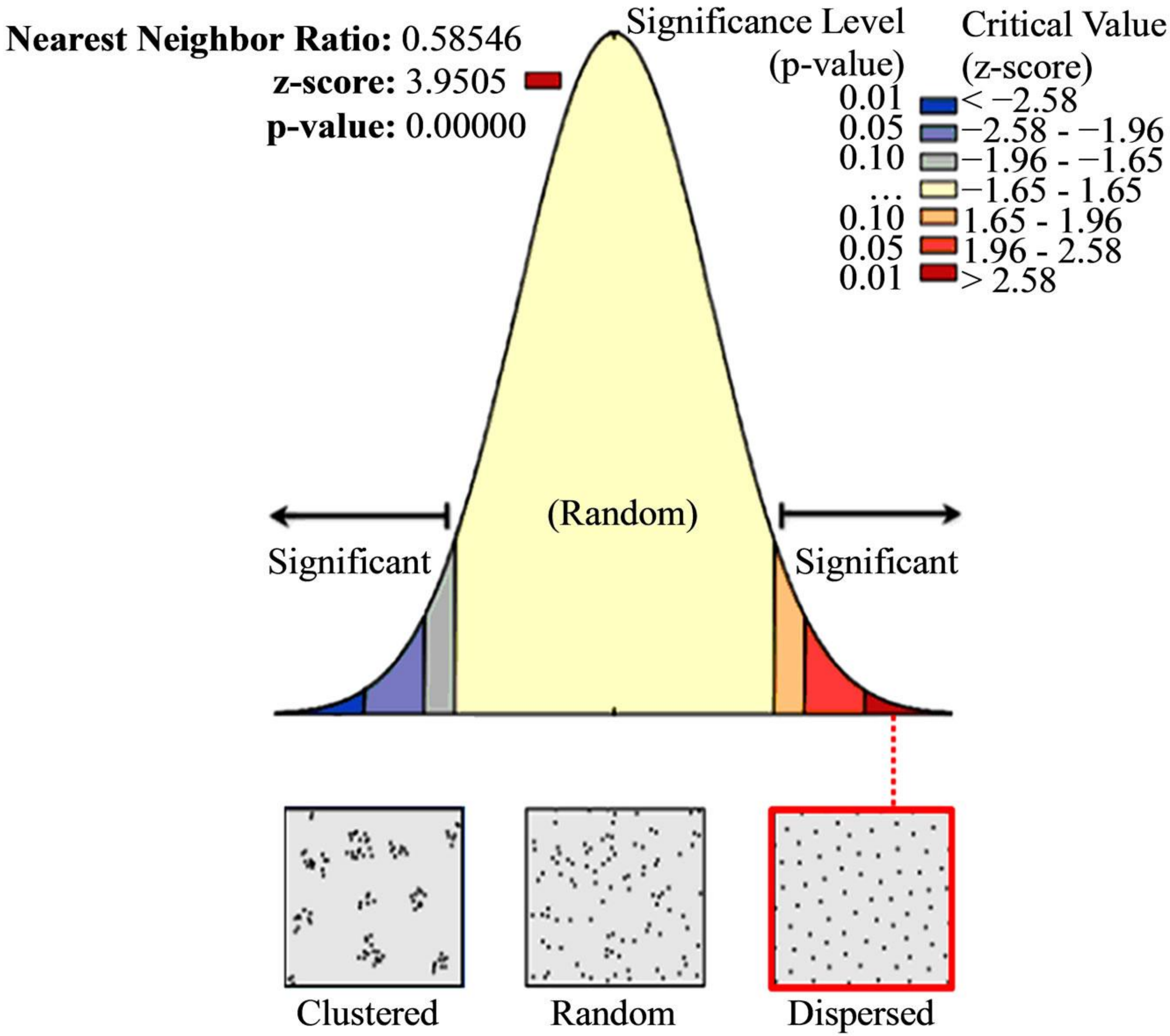
Standard Error of R-scale

- To describe the likelihood that any differences occur purely by chance
  - The calculated difference is relatively small compared to the std. error then the difference is statistically insignificant

  - The Std. Error for the obs. distance
    - $SE_r = 0.26136 / \sqrt{(n^2/A)}$
    - $Z_R = (r_{obs} - r_{exp}) / SE_r$
    - The difference will be statistically significant if

      $-1.96 < Z_R < 1.96$ (for α=0.05) two tailed

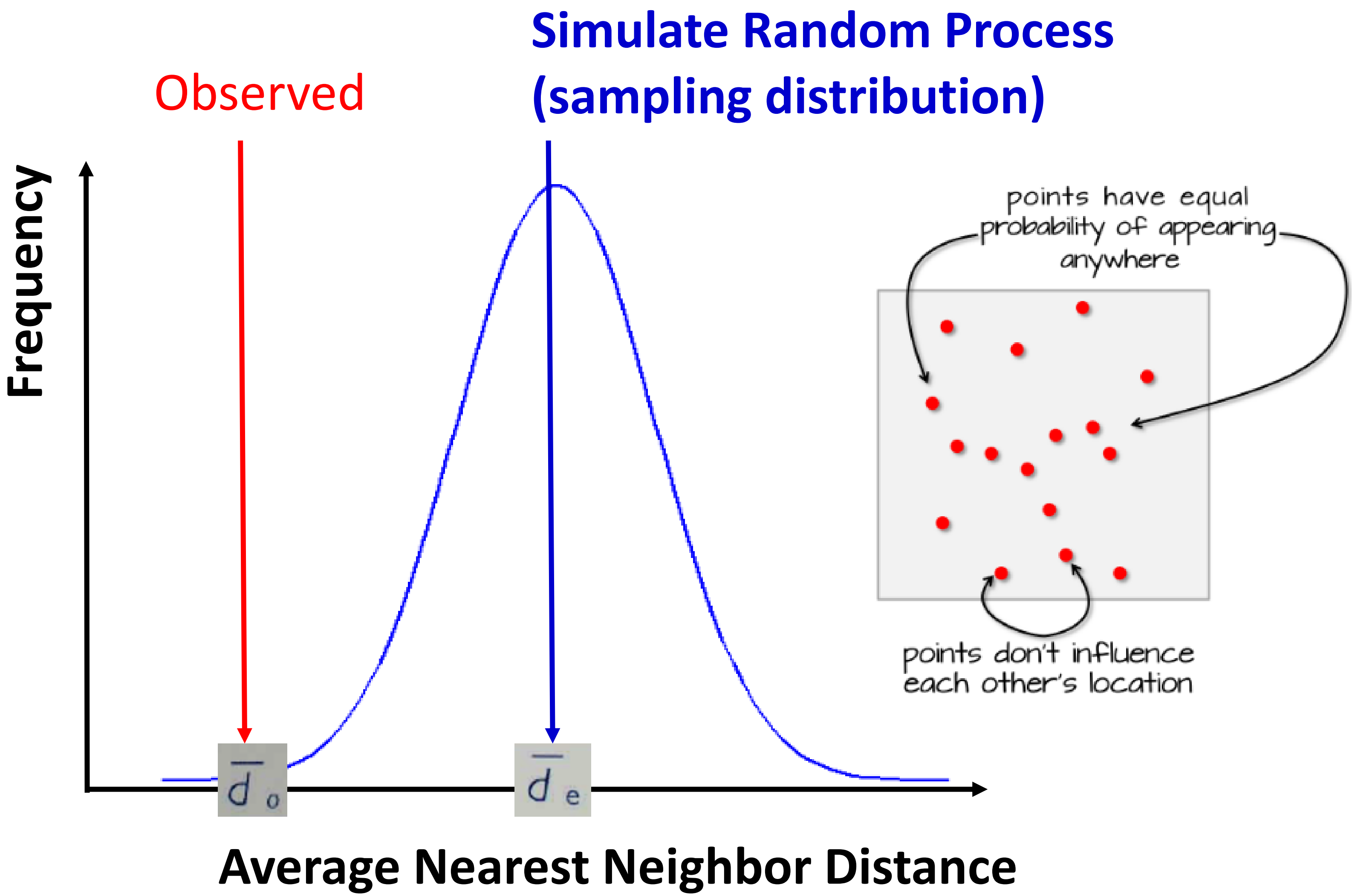
      (For single tailed 1.96 will changed as 1.645)


Nearest Neighbor Ratio: 0.58546
z-score: 3.9505
p-value: 0.00000
Significance Level (p-value)
Critical Value (z-score)
0.01 < −2.58
0.05 −2.58 - −1.96
0.10 −1.96 - −1.65
... −1.65 - 1.65
0.10 1.65 - 1.96
0.05 1.96 - 2.58
0.01 > 2.58
(Random)
Significant
Significant
Clustered
Random
Dispersed

Significance Test of NNA:
2. Monte Carlo Significance Test

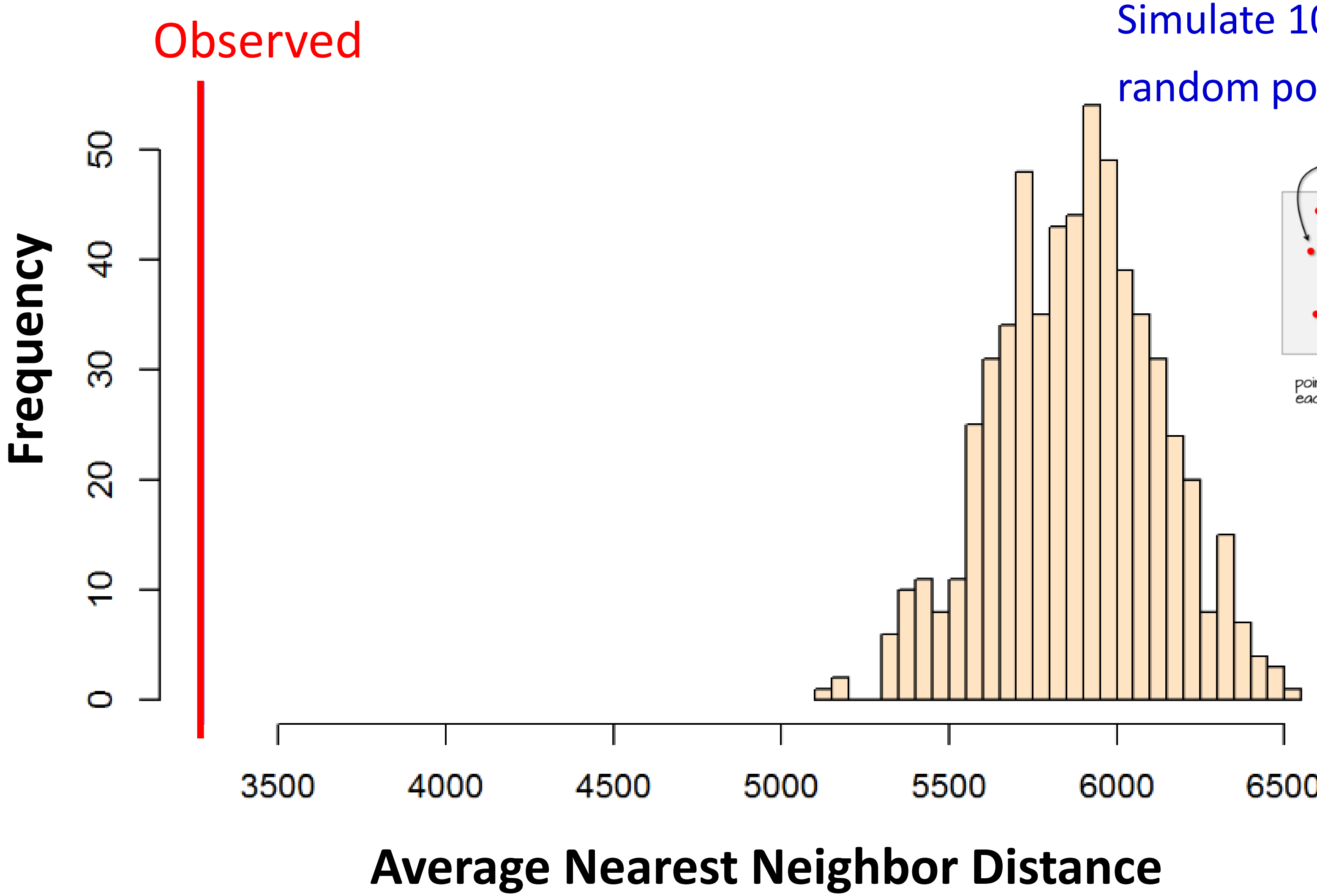
- The significance of any departures from CSR can be evaluated using simulated "confidence envelopes"
- Simulate many (eg. 100) spatial point processes
- Rank all the simulations
- Pull out the 5th and 95th values
- Plot these as the 95% confidence intervals

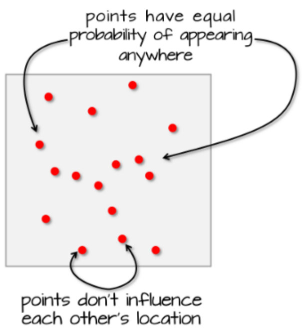
Monte Carlo Significance Test



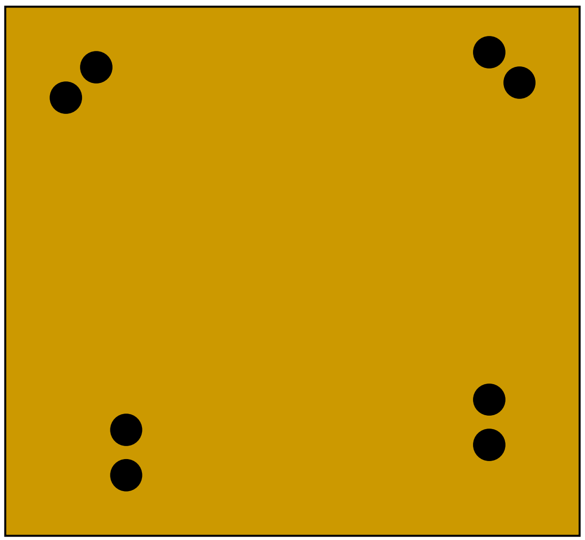
Monte Carlo Significance Test

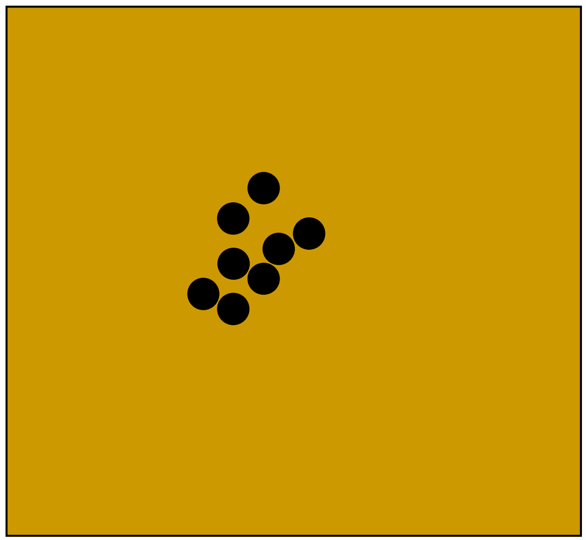
Simulate 1000 spatial random point processes



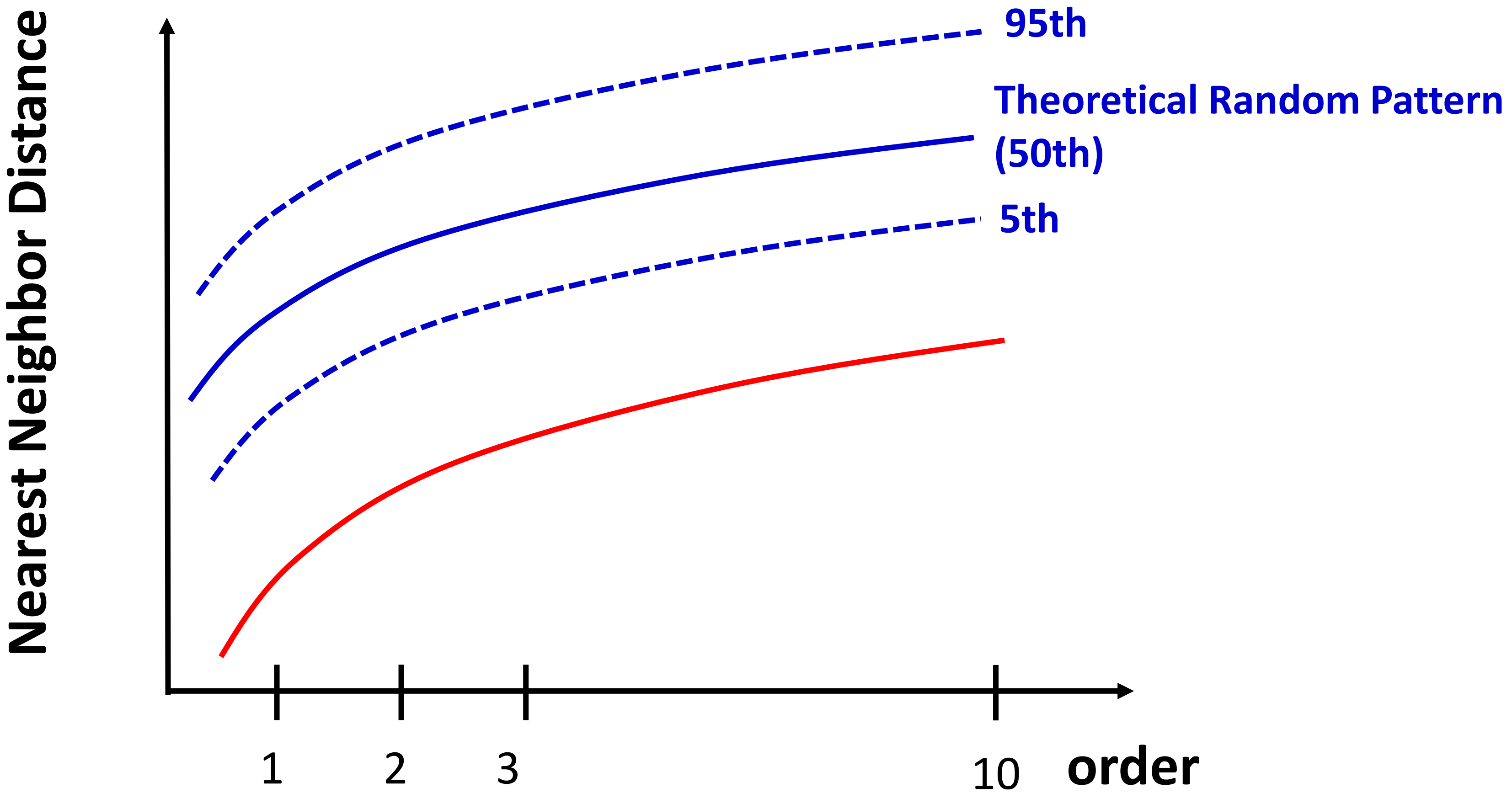


Why needs Higher Order NNA?

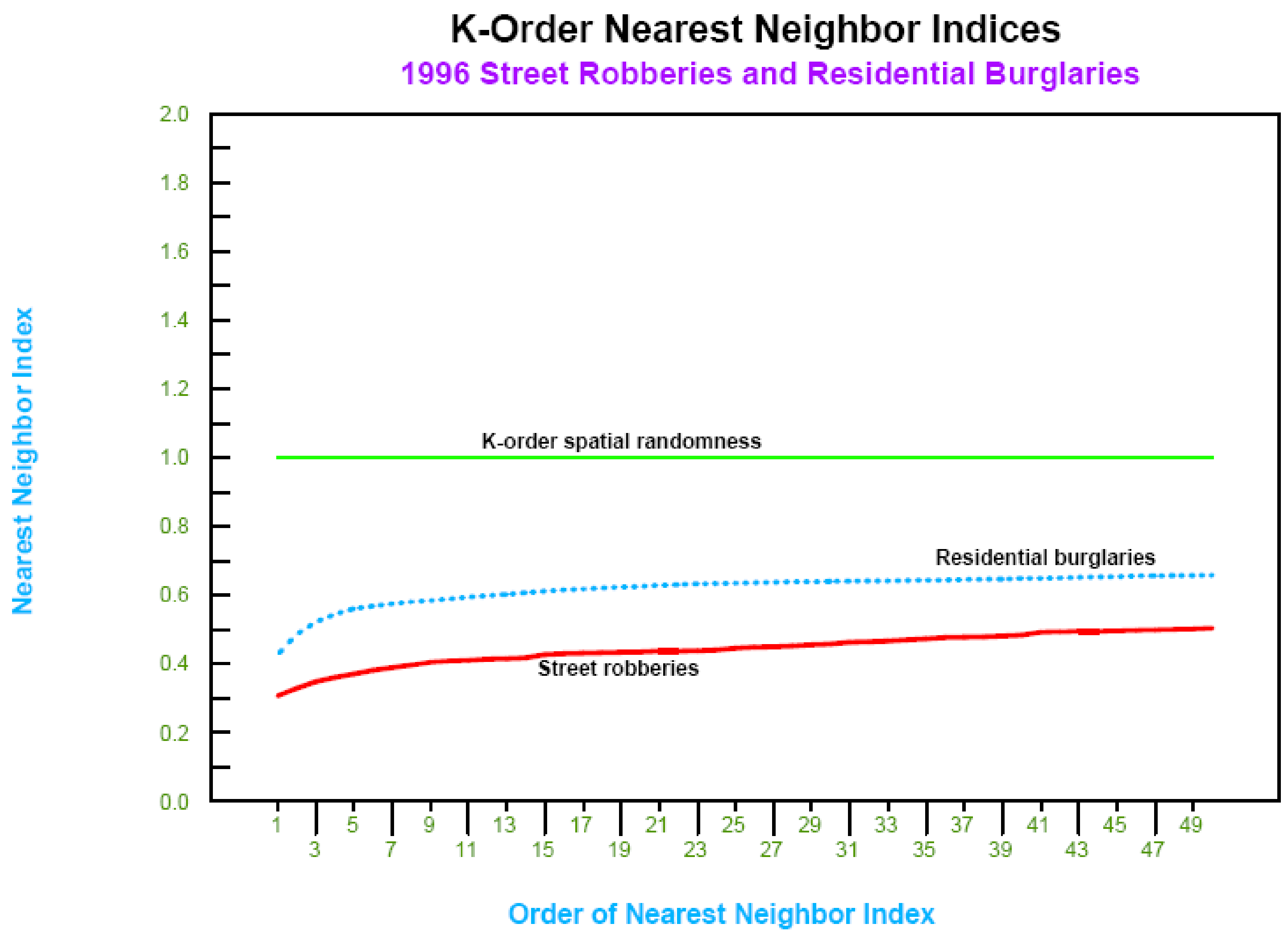




K-order Nearest Neighbor Distance



K-order Nearest Neighbor Indices: Example



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CALCULATING THE AVERAGE DISTANCE BETWEEN FEATURES

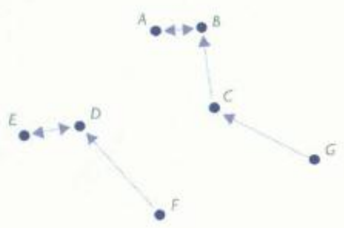
The nearest neighbor index is based on the research of ecologists Philip Clark and Frances Evans, who developed the method in the 1950s to quantify patterns in distributions of various plant species. In this method, the GIS finds the distance between each feature and its closest neighbor, then calculates the average (or mean) of these distances.

What the nearest neighbor index measures

The nearest neighbor index measures how similar the mean distance is to the expected mean distance for a hypothetical random distribution. The index is either the difference between the two or the ratio of the observed distance divided by the expected distance.

Calculating the observed mean distance

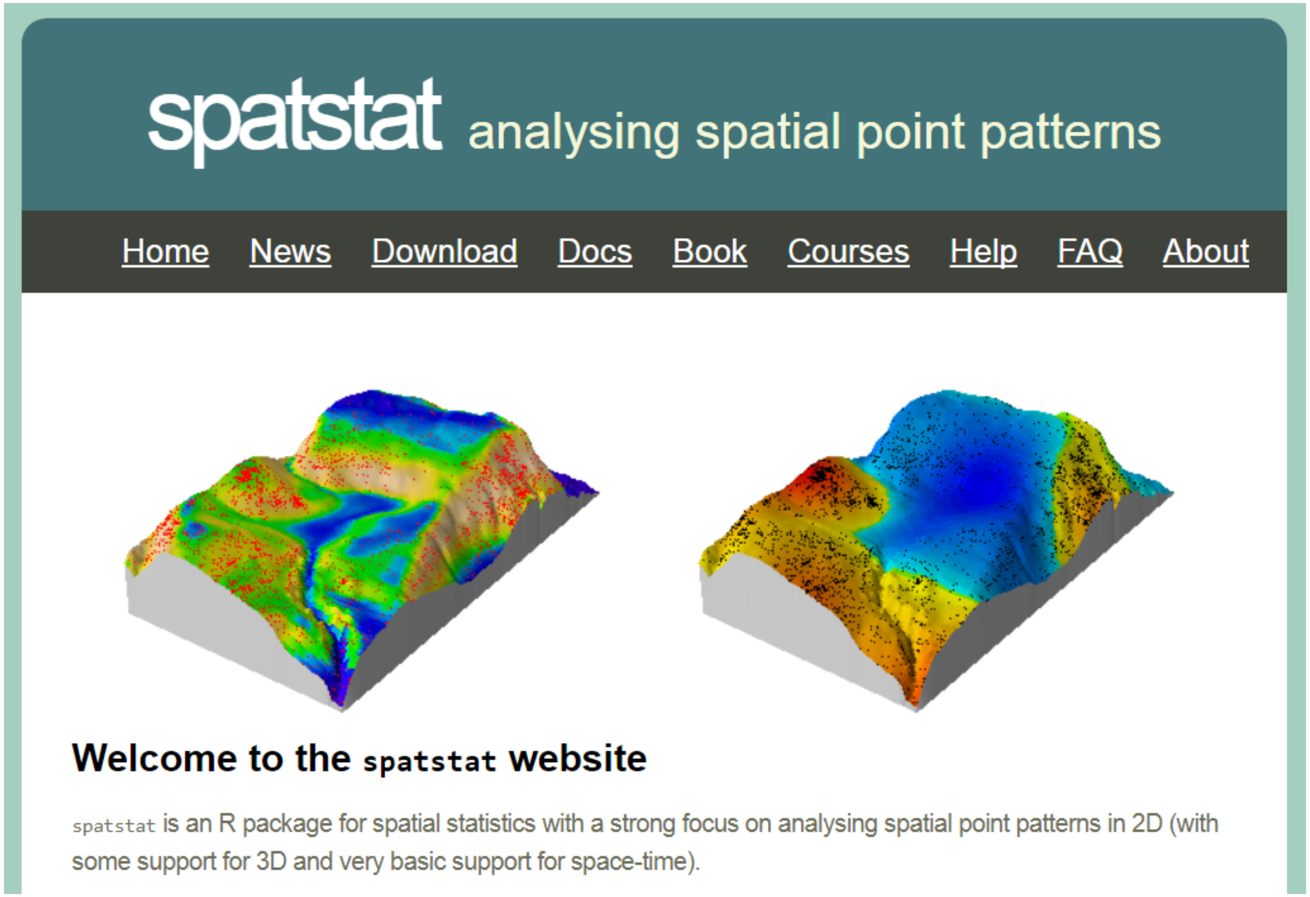
To get the distance from each feature to its nearest neighbor, the GIS essentially calculates the distance from each feature to all other features in the set, then finds the shortest distance, the nearest neighbor to the feature.



	A	B	C	D	E	F	G
A		988	2117	2494	3538	3858	4267
B	988		1725	3348	4308	4004	3601
C	2117	1725		2804	4034	2567	2309
D	2494	3148	2804		1196	2510	4897
E	3538	4308	4034	1196		3277	6034
F	3858	4004	2567	2510	3277		3433
G	4267	3601	2309	4897	6034	3433	

R Lab: Introducing spatstat package

http://spatstat.org/



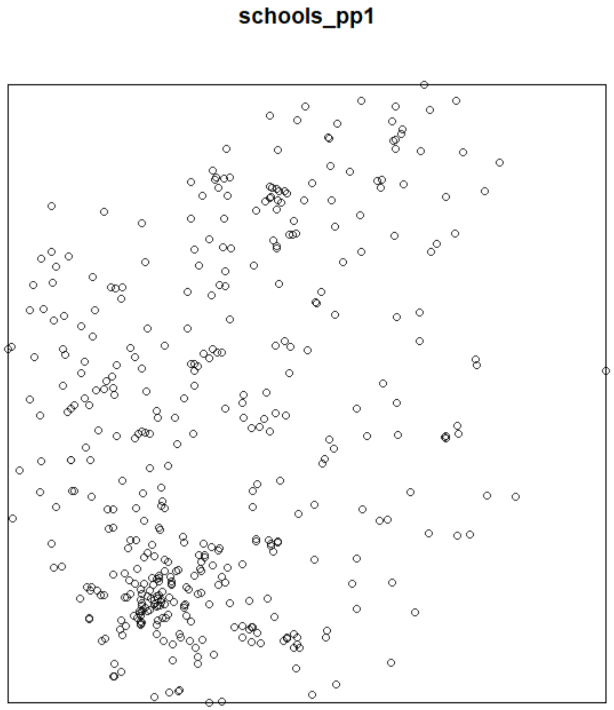
Spatstat: Introducing PPP format

```
schools_sf <- st_read("Schools.shp")

bnd <- st_bbox(schools_sf)
```

```
> bnd<-st_bbox(schools_sf)
> bnd
     xmin      ymin      xmax      ymax 
 155883.0 2535016.0  207754.2 2588604.0 
```

```
x.coor <- schools_sf$X_coor
y.coor <- schools_sf$Y_coor
x.range <- c(bnd[1],bnd[3])
y.range <- c(bnd[2],bnd[4])
schools_pp1 <- ppp(x.coor, y.coor, x.range, y.range)
```

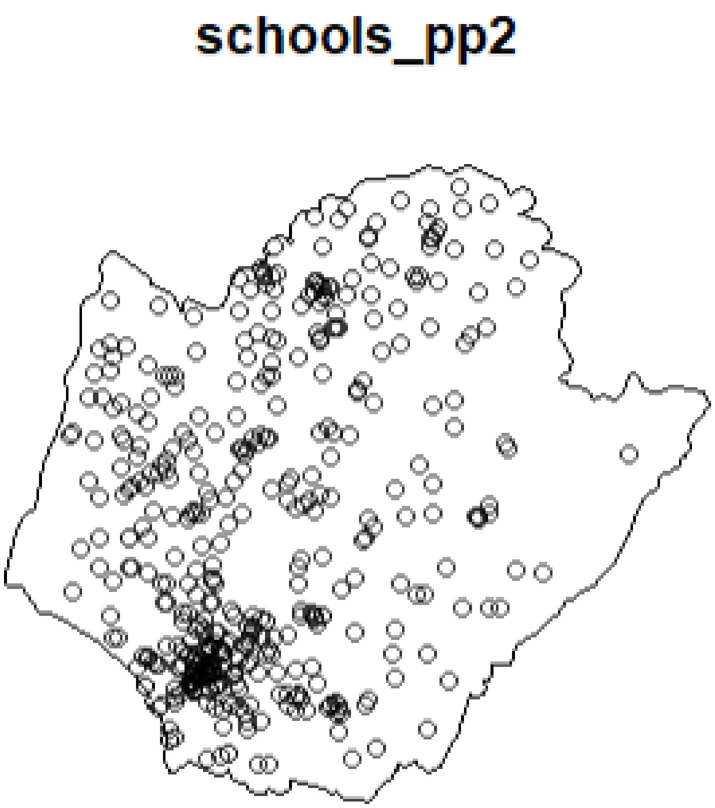


Spatstat: Introducing PPP format (2)

```
schools_sf <- st_read("Schools.shp")
TN_BND <- st_read("TainanCounty.shp")

coord <- st_coordinates(schools_sf)
Windows <- as.owin(TN_BND)
schools_pp2 <- as.ppp(coord, Windows)
```

PPP format: setting the boundary coordinates of a polygon



Average nearest neighbor distance -- using nndist()

spatstat (version 1.64-1)

nndist: Nearest neighbour distances

Description

Computes the distance from each point to its nearest neighbour in a point pattern. Alternatively computes the distance to the second nearest neighbour, or third nearest, etc.

```
# calculating the area
x<-x.range[2]-x.range[1]
y<-y.range[2]-y.range[1]
sqr.area<- x*y

nnd<-nndist(schools_pp1, k=1)
d1<-mean(nnd) # Tainan School
rd<- 0.5/sqrt(424/sqr.area)  # theoretical random pattern
r.scale <- d1/rd
```

R code: K-order Nearest Neighbor Distance

nndist(schools_pp1, k=1:20)

```
> nndist(schools_pp1, k=1:20)
```

School id

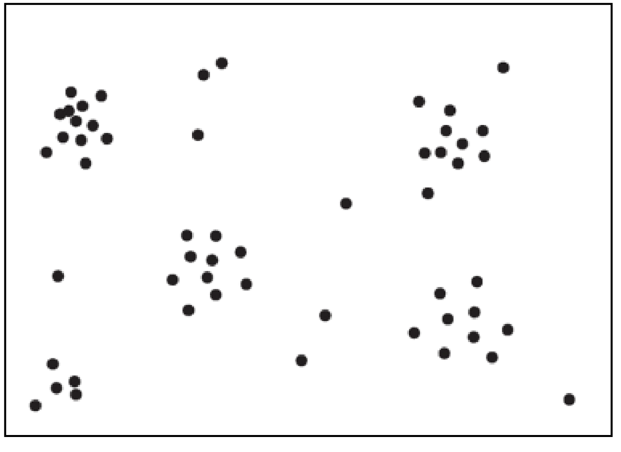
	dist.1	dist.2	dist.3	dist.4	dist.5
[1,]	3293.07708	3906.5673	4060.5085	4248.9399	5542.0873
[2,]	668.73292	2118.0747	2905.5489	3127.5107	3195.0453
[3,]	327.91614	362.3026	914.1177	1146.8717	1333.7450
[4,]	139.67384	306.3700	982.9686	984.7296	1329.9439
[5,]	139.67384	176.0771	865.0065	961.6888	1466.0069
[6,]	176.07710	306.3700	690.3840	1042.5882	1317.7148
[7,]	666.12329	690.3840	865.0065	982.9686	1518.2910
[8,]	1562.80945	1762.7531	1795.1202	2195.3224	2959.6642
[9,]	756.48794	1028.4560	1287.6734	1462.6867	1504.4356
[10,]	673.42023	849.8465	997.4824	1028.4560	1322.5454
[11,]	211.31029	1504.4356	1544.2539	1953.0921	2064.9741
[12,]	240.39489	1795.1202	1922.6266	2634.6808	2710.5818
[13,]	240.39489	1562.8095	2095.2726	2657.0145	2735.3344
[14,]	246.63068	464.6477	978.4555	1100.1906	1333.7450
[15,]	246.63068	678.1678	1220.2986	1324.5051	1572.0850
[16,]	54.12947	1925.9305	2217.4034	2273.8727	2284.3634
[17,]	54.12947	1978.5555	2166.0443	2225.3922	2327.4399
[18,]	1146.87169	1183.3835	1400.9277	1660.7438	1854.3505

k-order

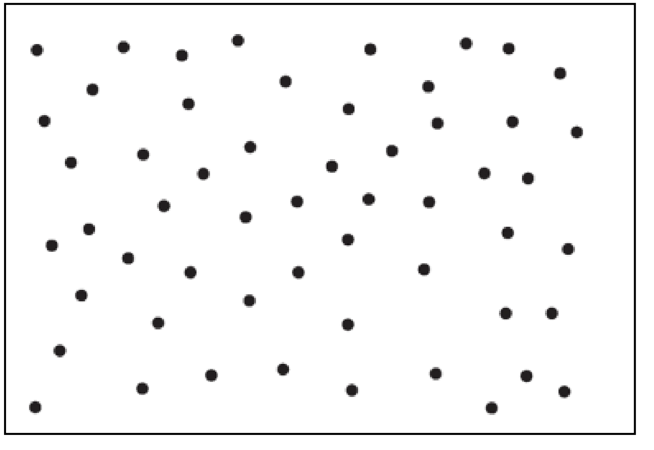
The Second-order Property: Distance-based Methods

- Nearest Neighbor Analysis (NNA)
- G Function: G(d)
- F Function: F(d)
- Ripley's K Functions: K (d) and L(d)

(PP1)



(PP2)



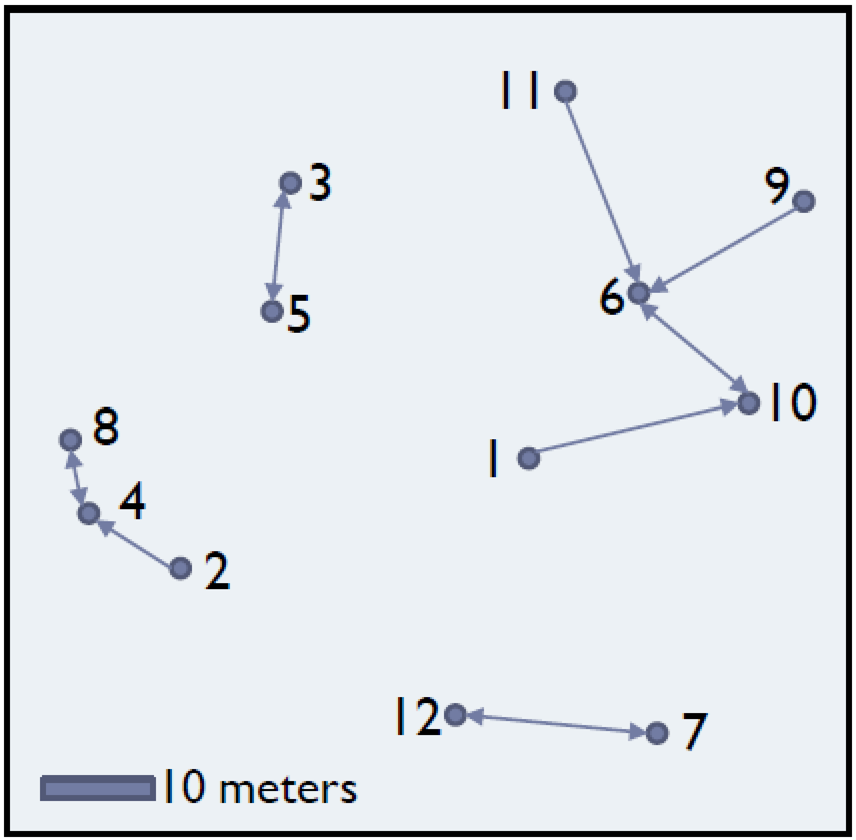
G Function: G(d)

- **The G function** is defined as the *cumulative frequency distribution* of the nearest-neighbor distances

$$G(d) = \frac{\#(d_{\min}(\mathbf{x}_i) < d)}{n}$$

G(d) gives the proportion (since the count is divided by n) of nearest-neighbor distances that are less than distance *d*.

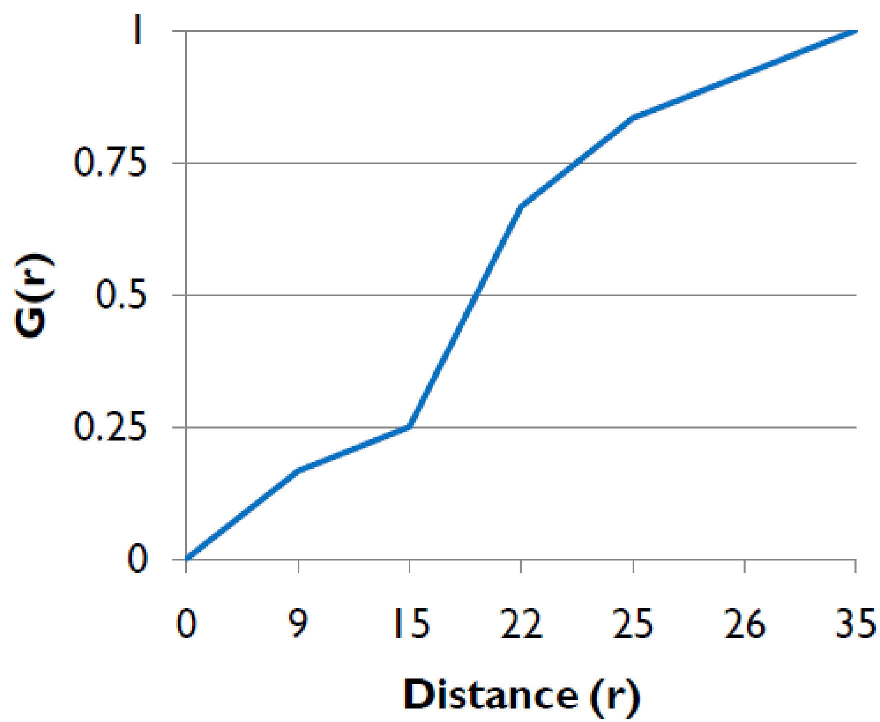
計算範例



Event	x	y	Nearest neighbor	r_{min}
1	66.22	32.54	10	25.59
2	22.52	22.39	4	15.64
3	31.01	81.21	5	21.14
4	9.47	31.02	8	24.81
5	30.78	60.10	3	9.00
6	75.21	58.93	10	21.14
7	79.26	7.68	12	21.94
8	8.23	39.93	4	9.00
9	98.73	42.53	6	21.94
10	89.78	42.53	6	21.94
11	65.19	92.08	6	34.63
12	54.46	8.48	7	24.81

計算範例：產生 G Function, G(r)

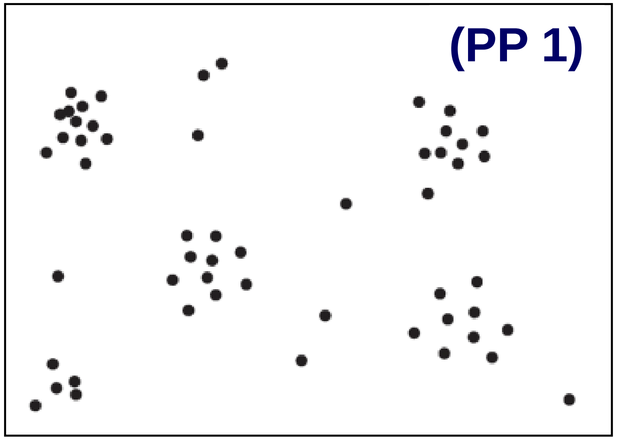
Event	x	y	Nearest neighbor	r_{min}
1	66.22	32.54	10	25.59
2	22.52	22.39	4	15.64
3	31.01	81.21	5	21.14
4	9.47	31.02	8	24.81
5	30.78	60.10	3	9.00
6	75.21	58.93	10	21.14
7	79.26	7.68	12	21.94
8	8.23	39.93	4	9.00
9	98.73	42.53	6	21.94
10	89.78	42.53	6	21.94
11	65.19	92.08	6	34.63
12	54.46	8.48	7	24.81

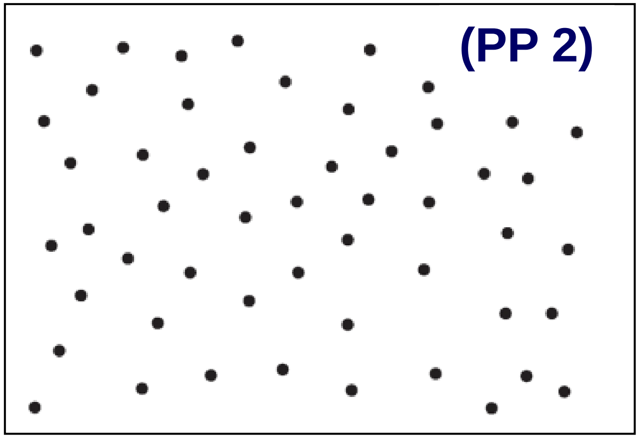


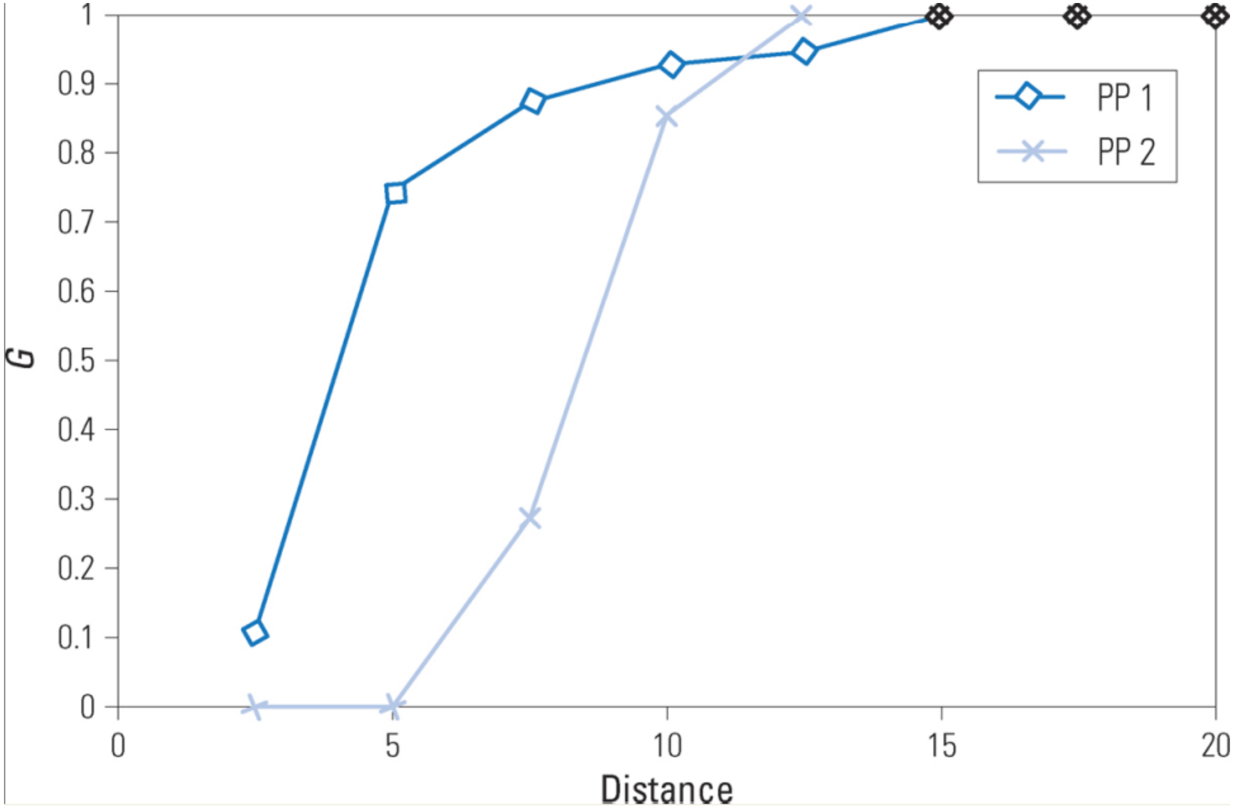
G Function的解讀

- The shape of G-function tells us the way the events are spaced in a point pattern
- **Clustered**：G increases rapidly at short distance
- **Uniform**：G increases slowly up to distance where most events spaced, then increases rapidly

範例





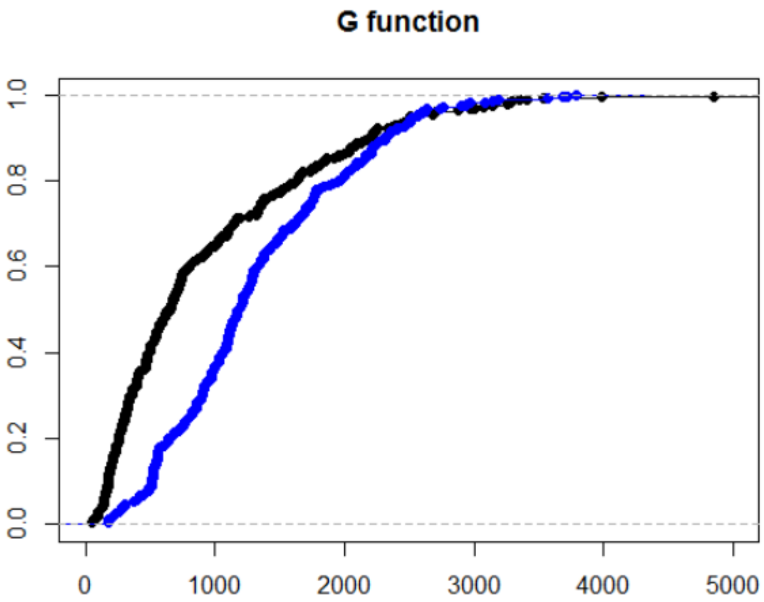


R code: G(d) Function

```
nnd<-nndist(School.ppp, k=1)
G = ecdf(nnd)
plot(G, main="G function", xlim=c(0,5000))
```

```
TN.Windows<-owin(xrange=x.range, yrange=y.range)
nn1<-rpoint(424, win=TN.Windows)
plot(nn1)

nnd1<-nndist(nn1, k=1)
G1 = ecdf(nnd1)
lines(G1,col='blue')
```



How do we examine significance？

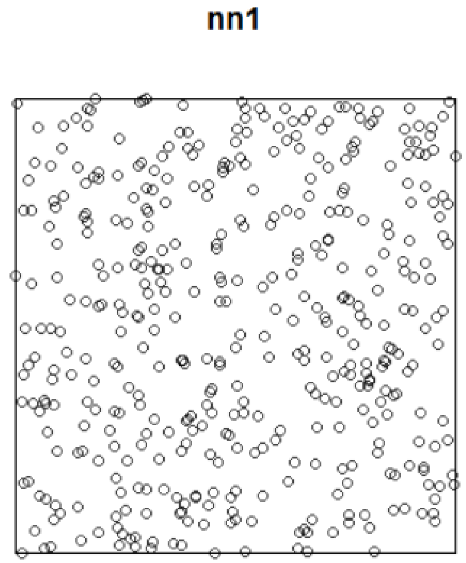
Significant departure from Complete Spatial Randomness (CSR)

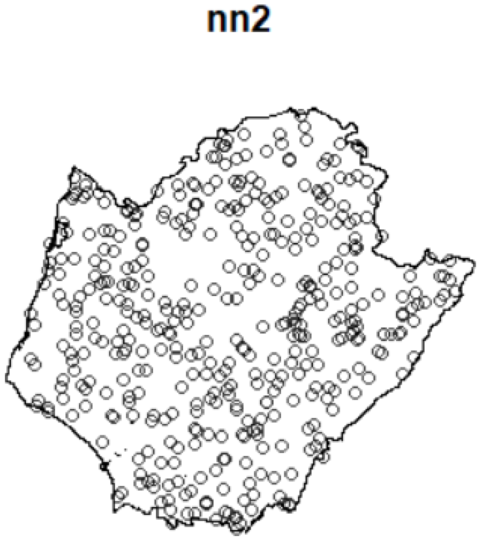
- The significance of any departures from CSR (either clustering or uniform) can be evaluated using simulated “confidence envelopes”
- Simulate many (eg. 1000) spatial point processes and estimate the G function for each of these
- Rank all the simulations
- Pull out the 5th and 95th G(r) values
- Plot these as the 90% confidence intervals

R Lab: Generating Random Points: rpoint()

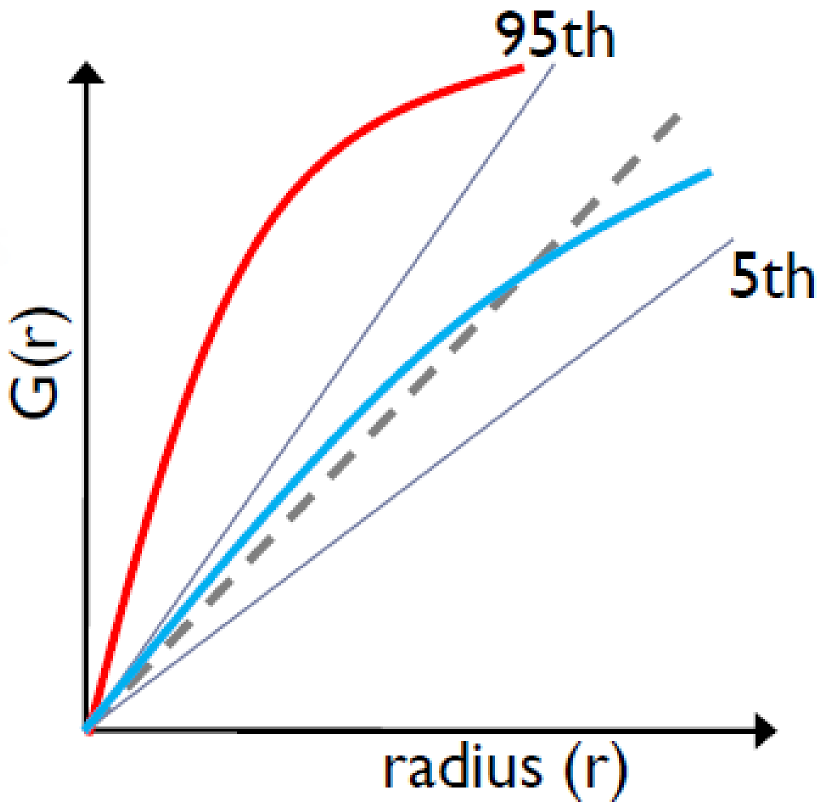
```
TN.Windows<-owin(xrange=x.range, yrange=y.range)
nn1<-rpoint(424, win=TN.Windows)
```

```
nn2<- rpoint(415, win = PTS_bnd)
plot(nn2, pch=16)
```





顯著性檢定的圖示



單尾檢定 (one-tailed test)

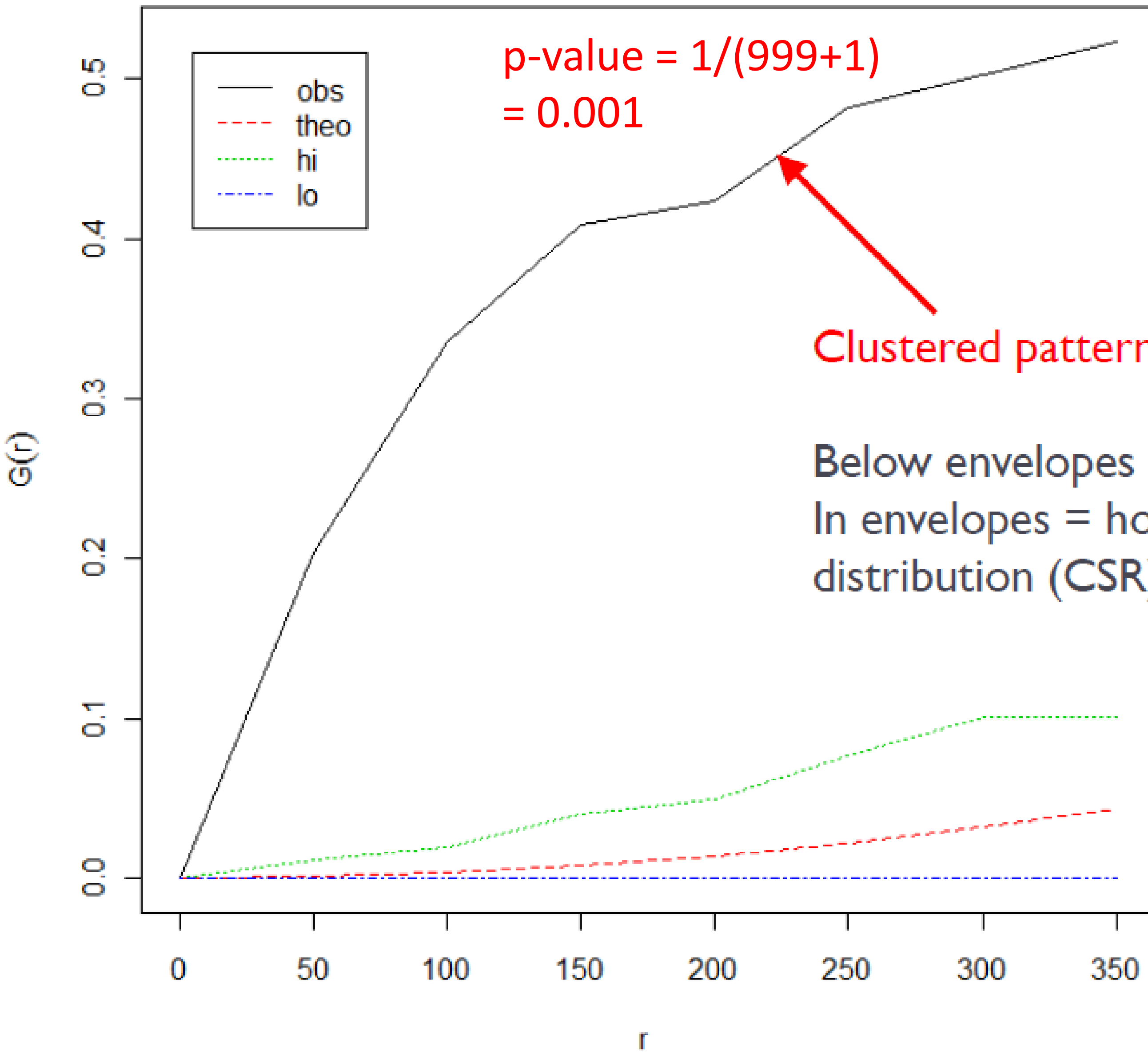
H0: G(r) <= Gr(r) of random 95th percentile

H1: G(r) > Gr(r) of random 95th percentile

If reject H0, it can conclude that

It shows a **statistically significant clustering** pattern (p-value < 0.05)

範例

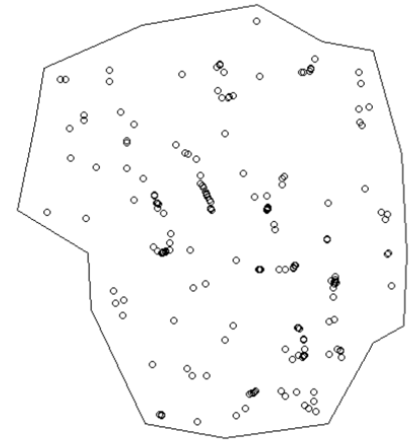


p-value = 1/(999+1)
= 0.001

Clustered pattern (above the envelopes)

Below envelopes = regular pattern
In envelopes = homogeneous
distribution (CSR)

999 simulation runs
of random points

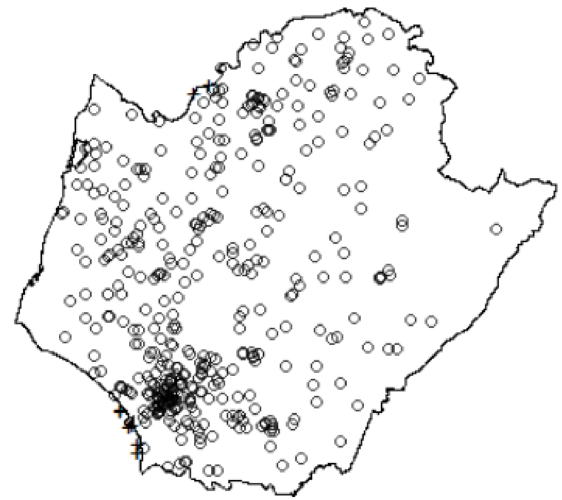


本週實習: Analysis of Nearest-Neighbor Distances

- 圖資：
  - 台南市學校 schools.shp
  - 台南市邊界 TainanCounty.shp
- 分析方法：

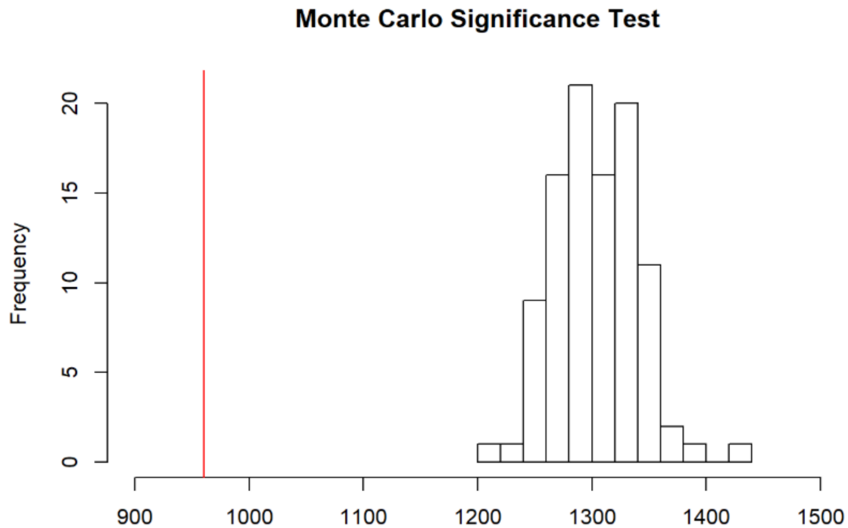
  （以行政區範圍為研究區邊界）
  - 1. Nearest Neighbor Analysis
  - 2. K-order Nearest Neighbor Indices
  - 3. G Function
- 用Monte Carlo Simulation檢定統計顯著性

School.ppp2

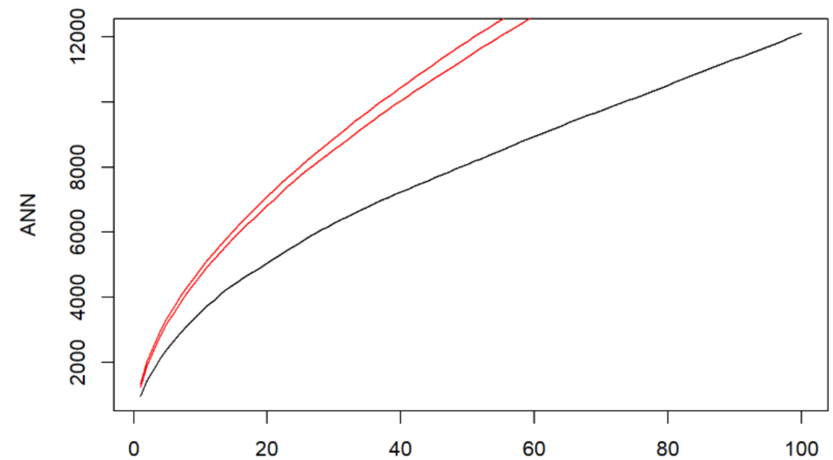


参考答案

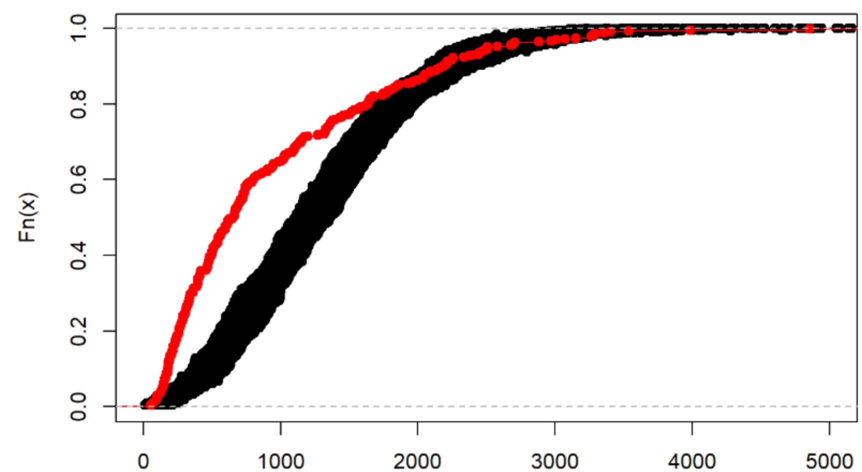
Nearest Neighbor Analysis



K-order Nearest Neighbor Distances



G Function



實習教學影片、簡報檔與程式碼

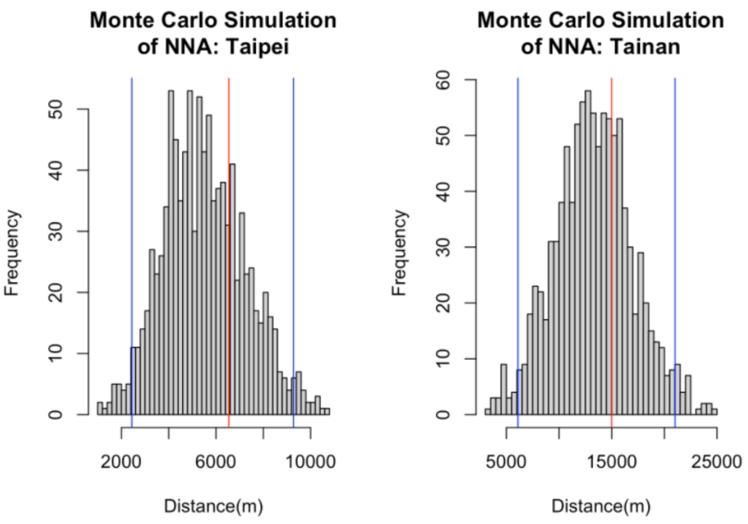
https://wenlab501.github.io/GEOG2017/

本週作業

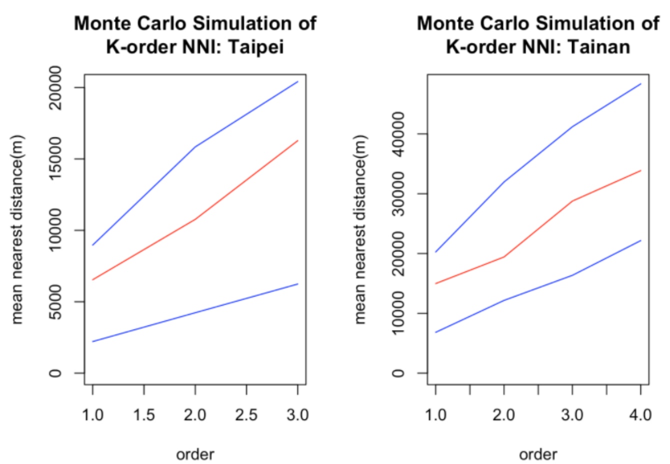
- 利用課堂提供的資料，利用Nearest-Neighbor Distances，比較任兩個縣市信仰「觀音菩薩」的村落型祭祀圈的寺廟空間群聚特性，並討論之。（包括：Nearest Neighbor Analysis, K-order Nearest Neighbor Distances(or Indices), and G(d) Function）

參考答案：台北市 vs. 台南市

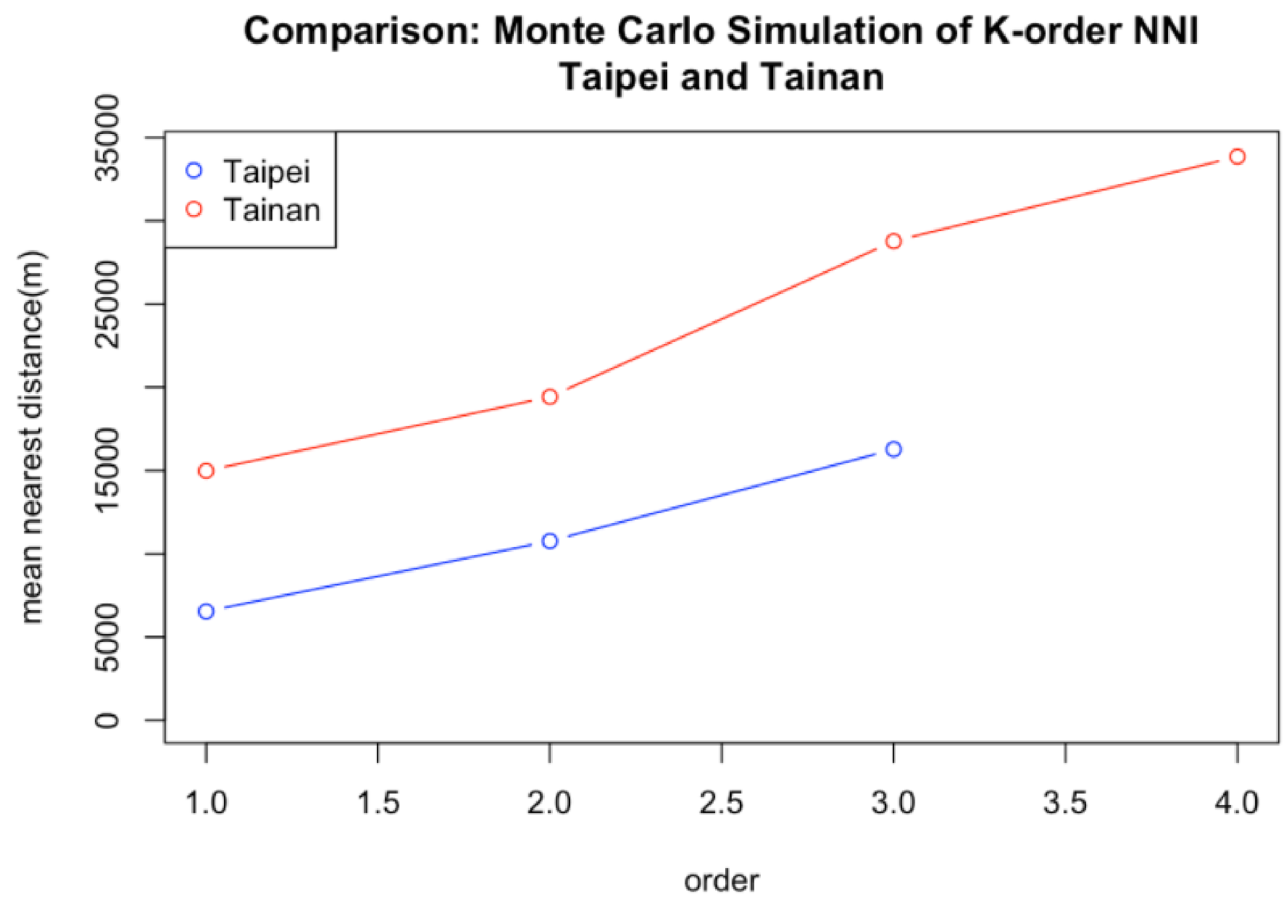
Nearest Neighbor Analysis



K-order Nearest Neighbor Distances



參考答案：台北市 vs. 台南市



參考答案：台北市 vs. 台南市

G(d) Function