# Describing a Point Pattern

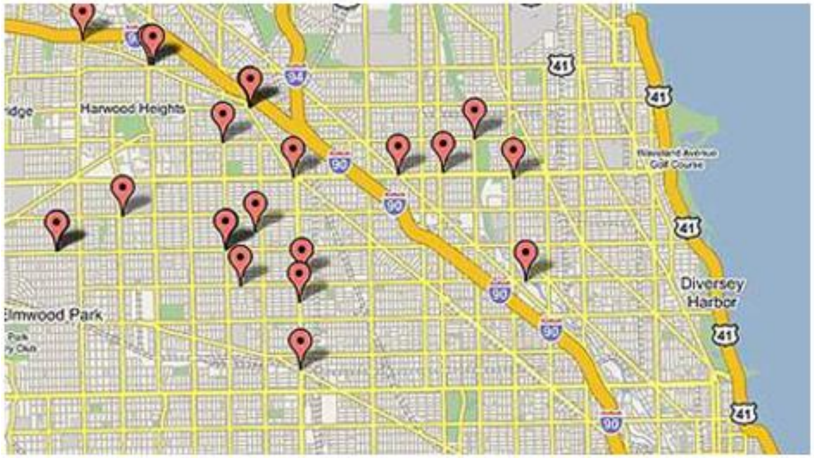
授課教師：溫在弘
E-mail: wenthung@ntu.edu.tw

# 期中考試相關規定與說明

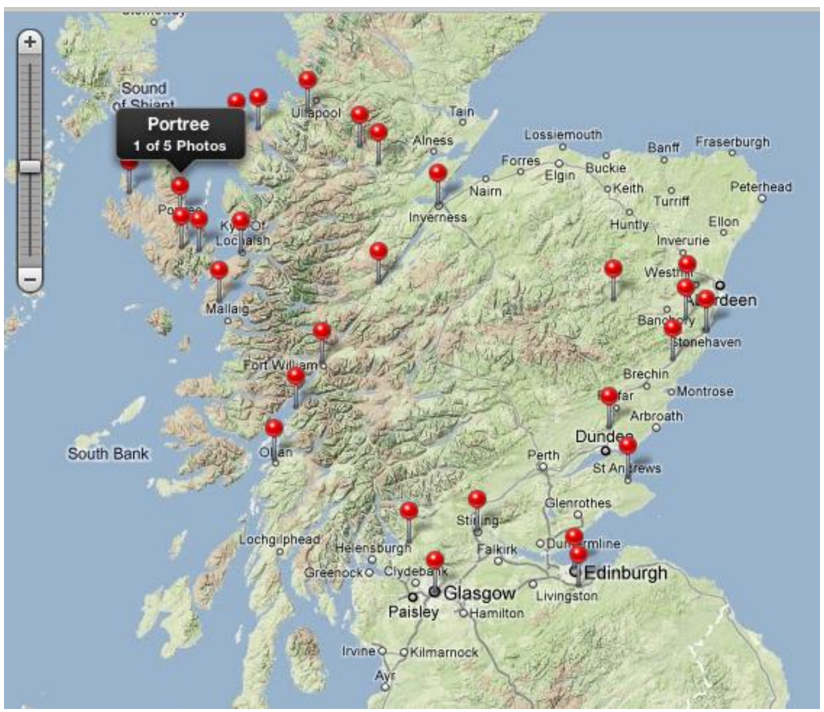
- 2025/04/07 下午2:30 – 5:30
- 進行方式：紙筆測驗（可查閱講義、筆記等）
- 若當天無法考試，需出示公假或醫師證明，並主動聯繫助教安排補考
- 分數計算 (總成績30%)：紙筆測驗(100分) + 兩次課程小考

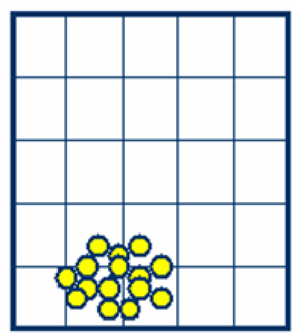
  若超過100分，仍以100分計算。
- 違反以下則視同作弊（違反則學期等第不及格）：
  - 使用任何電子設備（包含手機、平版或筆電等）
  - 任何形式的交談、資訊共用或交換

# The Era of Geotagging: What's the Next?

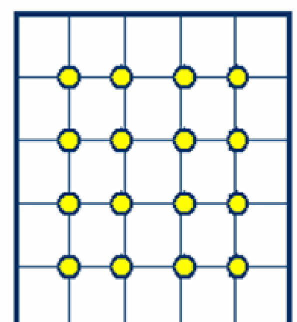



Flickr: pixeljones

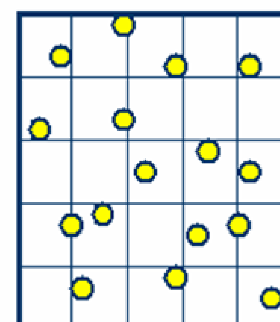


(a) Clustering



(b) Dispersion/Uniform



(c) Random

# Spatial Point Pattern Analysis

- Measuring Geographic Distribution
  - Spatial Mean & Standard Distance
  - Spatial Median
  - Standard Deviational Ellipse
- Visualizing Point Density
  - Kernel Density Estimation (KDE)

**Exploratory Analysis:**

Descriptive Statistics

- Analyzing ***Global*** Patterns
  - Quadrat Analysis
  - Nearest Neighbor Methods
  - Distance-based Methods

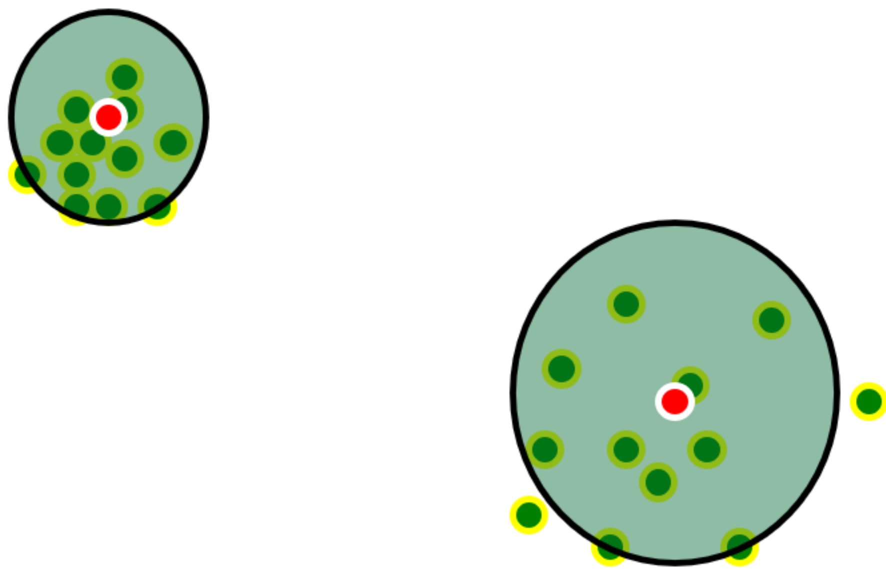
**Confirmatory Analysis:**

Inferential Statistics

# Issues of Point Pattern Description

- ***WHAT*** are the centrality & dispersion of a set of points data.
- ***WHY*** mapping the centrality & dispersion.
- ***HOW*** to measure the centrality & dispersion.

# Why mapping the centrality & dispersion of a set of points

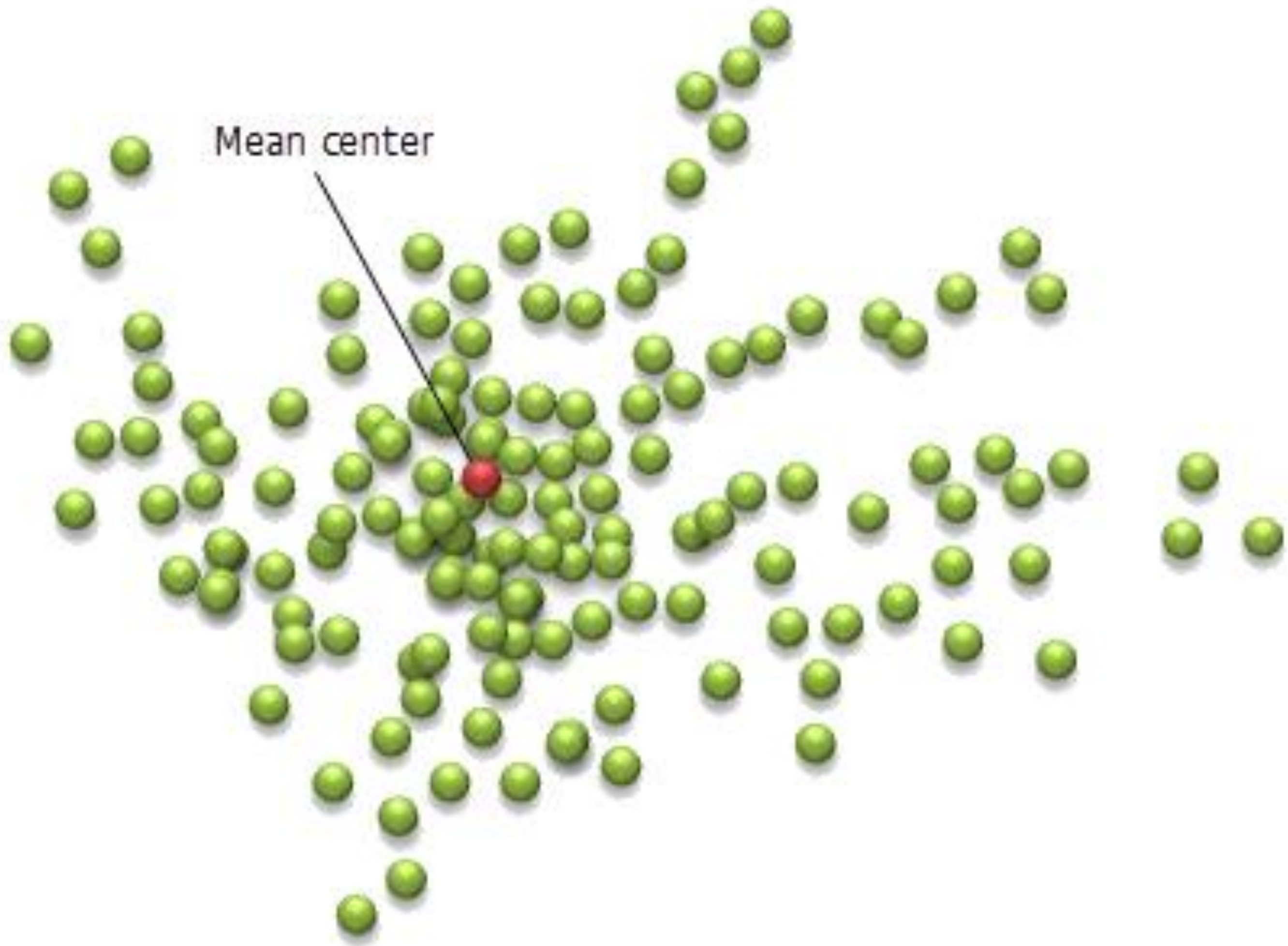


# Measures of Centrality

- Mean Center $$(x_{mc}, y_{mc}) = (\frac{\sum x_i}{n}, \frac{\sum y_i}{n})$$
- Weighted mean center $$(x_{wmc}, y_{wmc}) = (\frac{\sum w_i x}{\sum w_i}, \frac{\sum w_i y_i}{\sum w_i})$$
- Median Center

  1. **Center of Minimum Distance**
     - Euclidean distance / Manhattan distance
  2. **As the region is divided into 4 quadrants with equal numbers of data point**
- Weighted median center

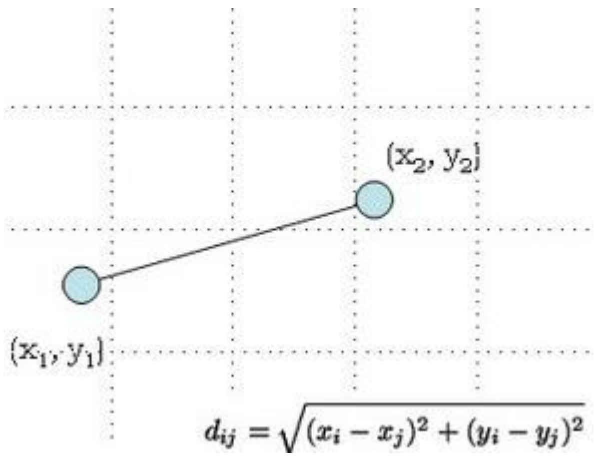
# Mean Center



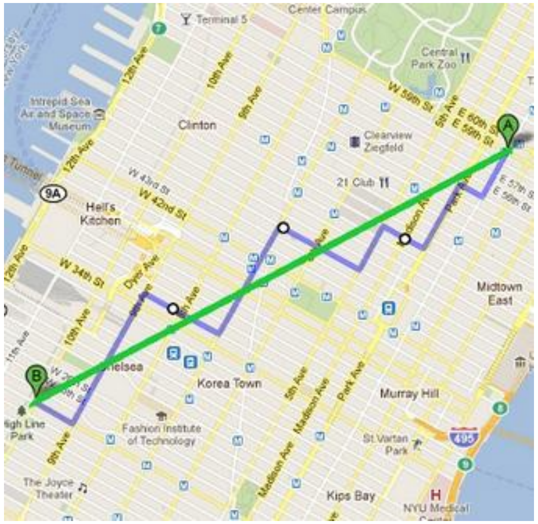
# Median Center

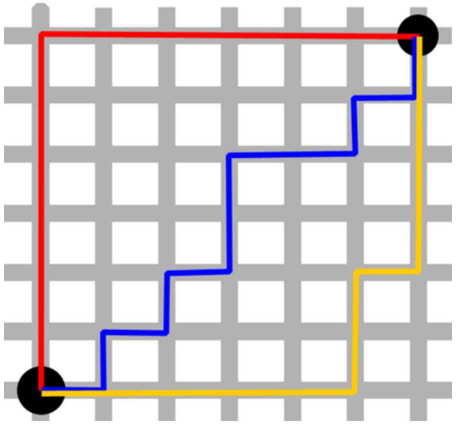
*Minimize* $\sum_i$ $d_i^t = \sqrt{(X_i - X^t)^2 + (Y_i - Y^t)^2}$

**Euclidean distance**
**歐幾里得距離**

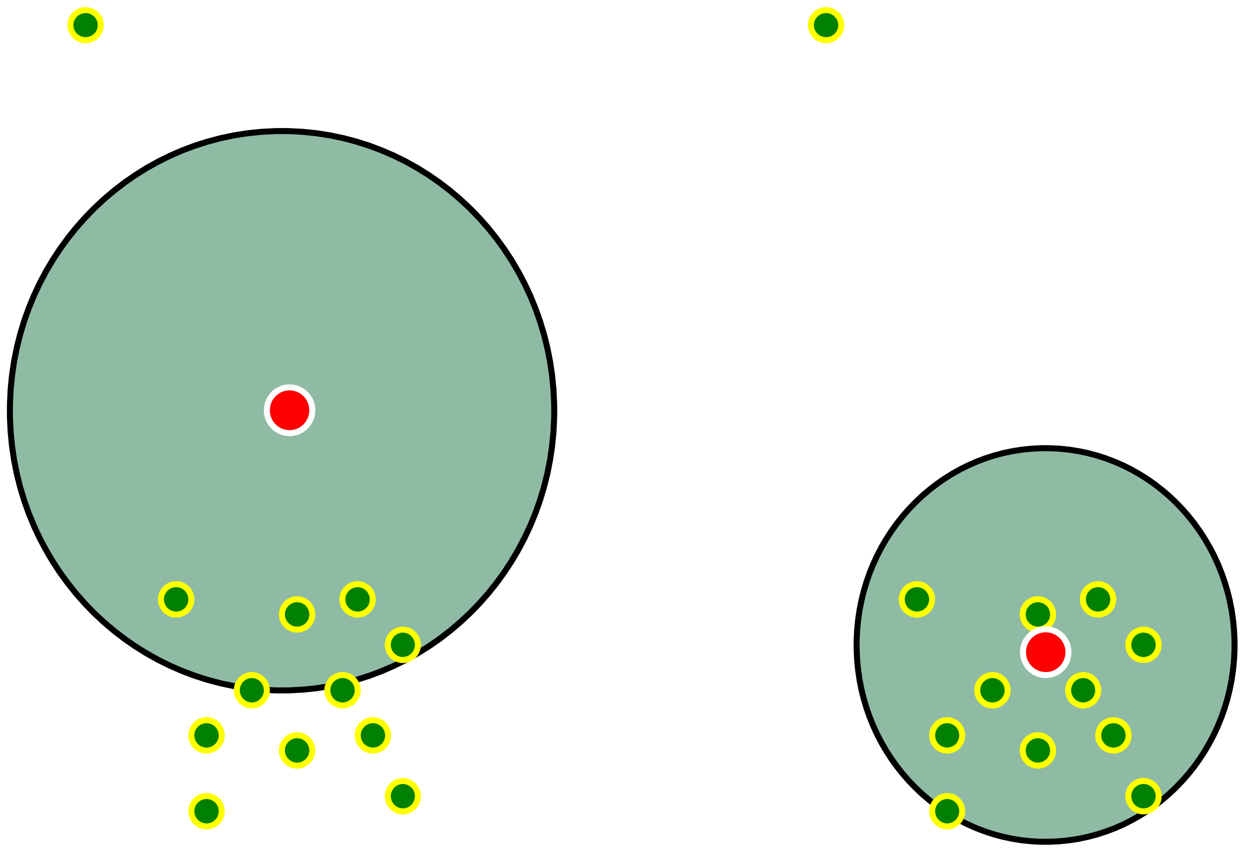



**Manhattan distance**
**曼哈頓距離**

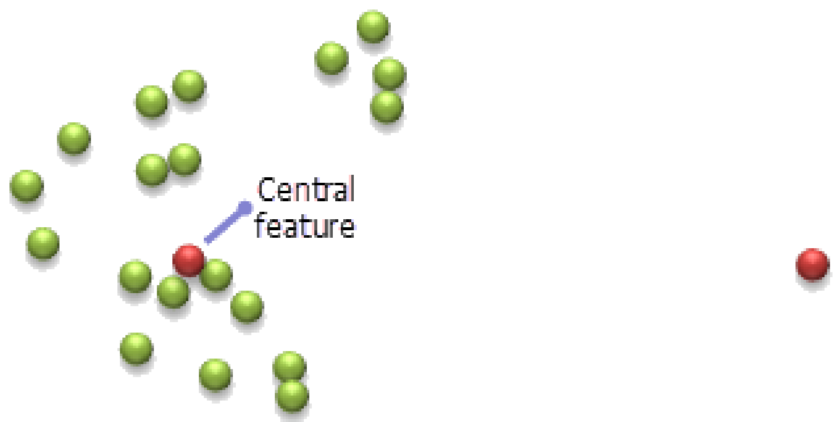


# Mean Center vs. Median Center



# Central Feature



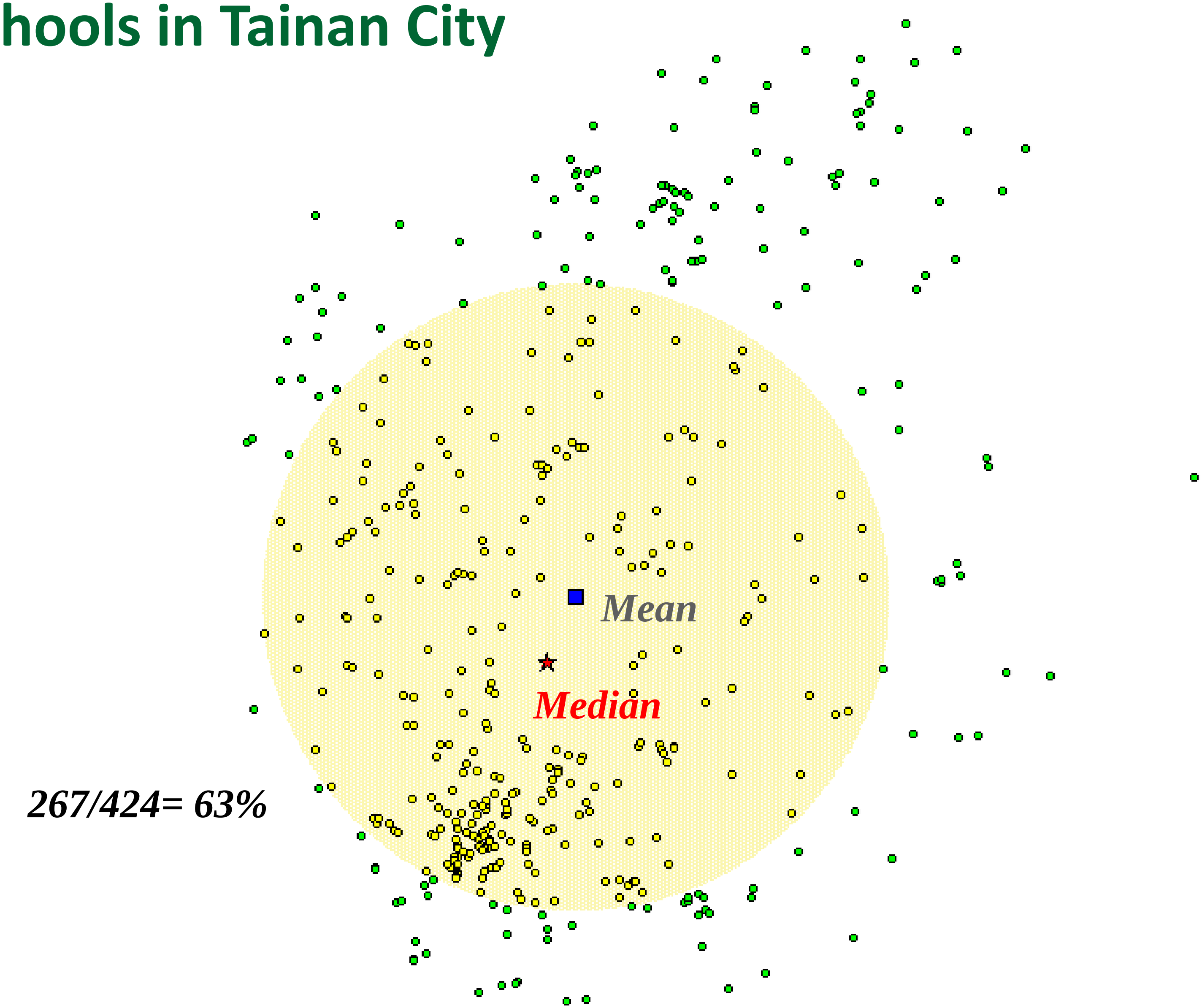

| | OBJECTID | Shape | Name |
|---|---|---|---|
| | 1 | Point | A |
| | 2 | Point | B |
| | 3 | Point | C |
| | 4 | Point | D |
| | 5 | Point | E |
| | 6 | Point | F |
| | 7 | Point | G |
| | 8 | Point | H |
| | 9 | Point | I |
| | 10 | Point | J |
| | 11 | Point | K |
| | 12 | Point | L |

| | OBJECTID | Shape | Name |
|---|---|---|---|
| ▶ | 1 | Point | G |

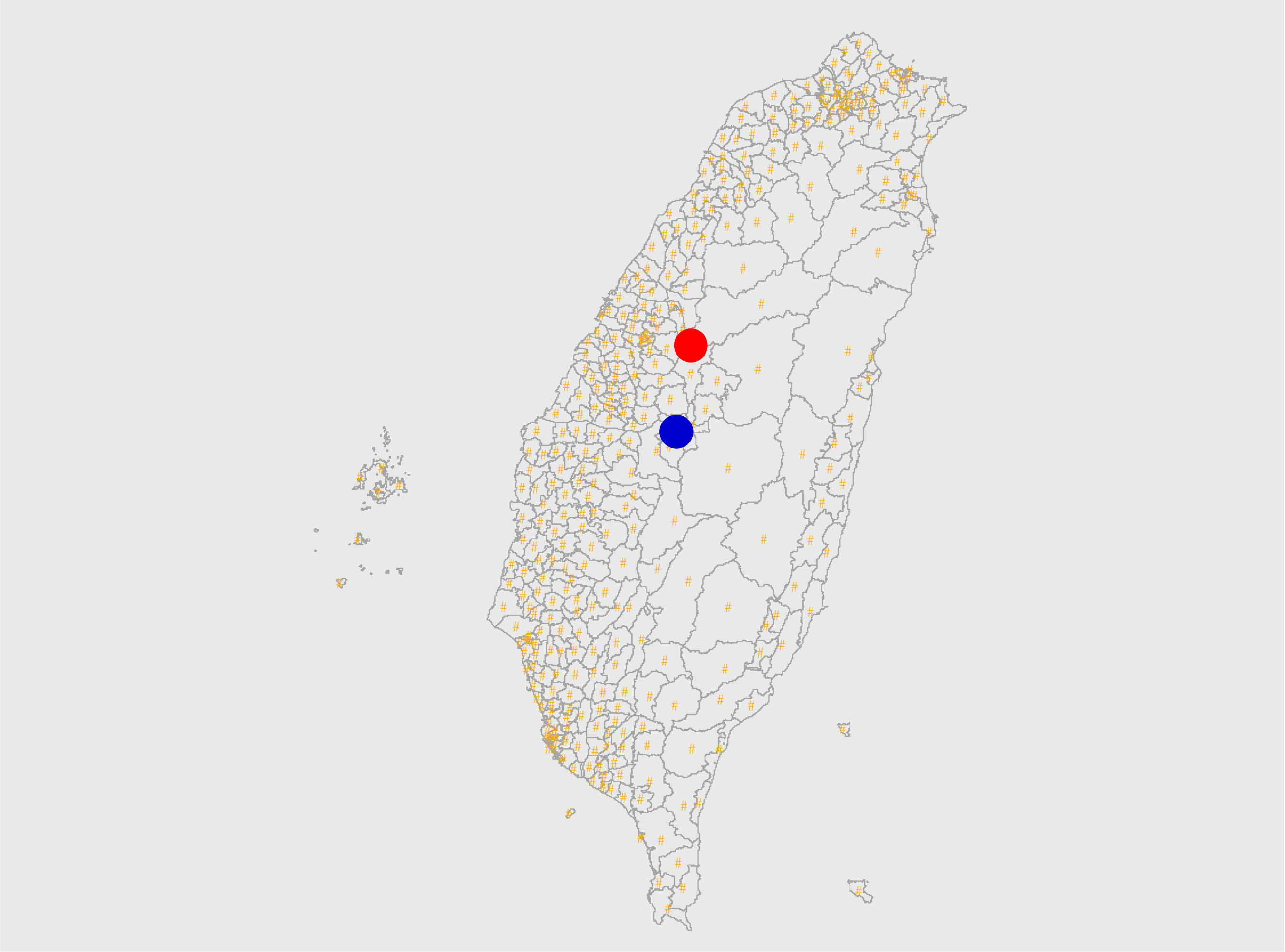
# Summary

| Center | What it represents | What it's good for |
|---|---|---|
| Mean | The average x-coordinate and average y-coordinate for all features in the study area | Tracking changes or comparing distributions |
| Median | The x,y coordinate having the shortest distance to all features in the study area | Finding the most accessible location |
| Central feature | The feature having the shortest total distance to all other features in the study area | Finding the most accessible feature |

# Demo: Schools in Tainan City



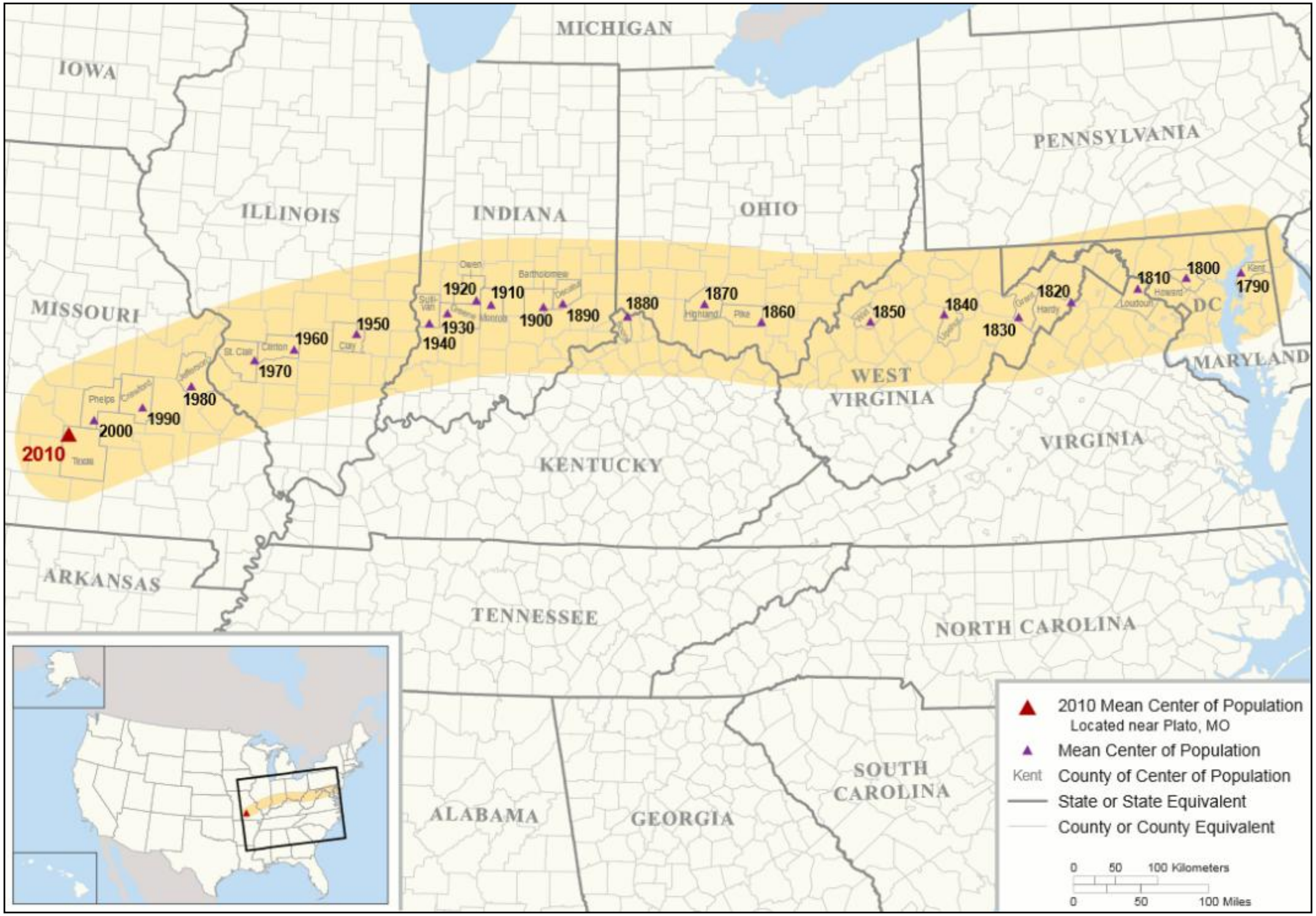
# Demo: Geographic Center (weighted by population)



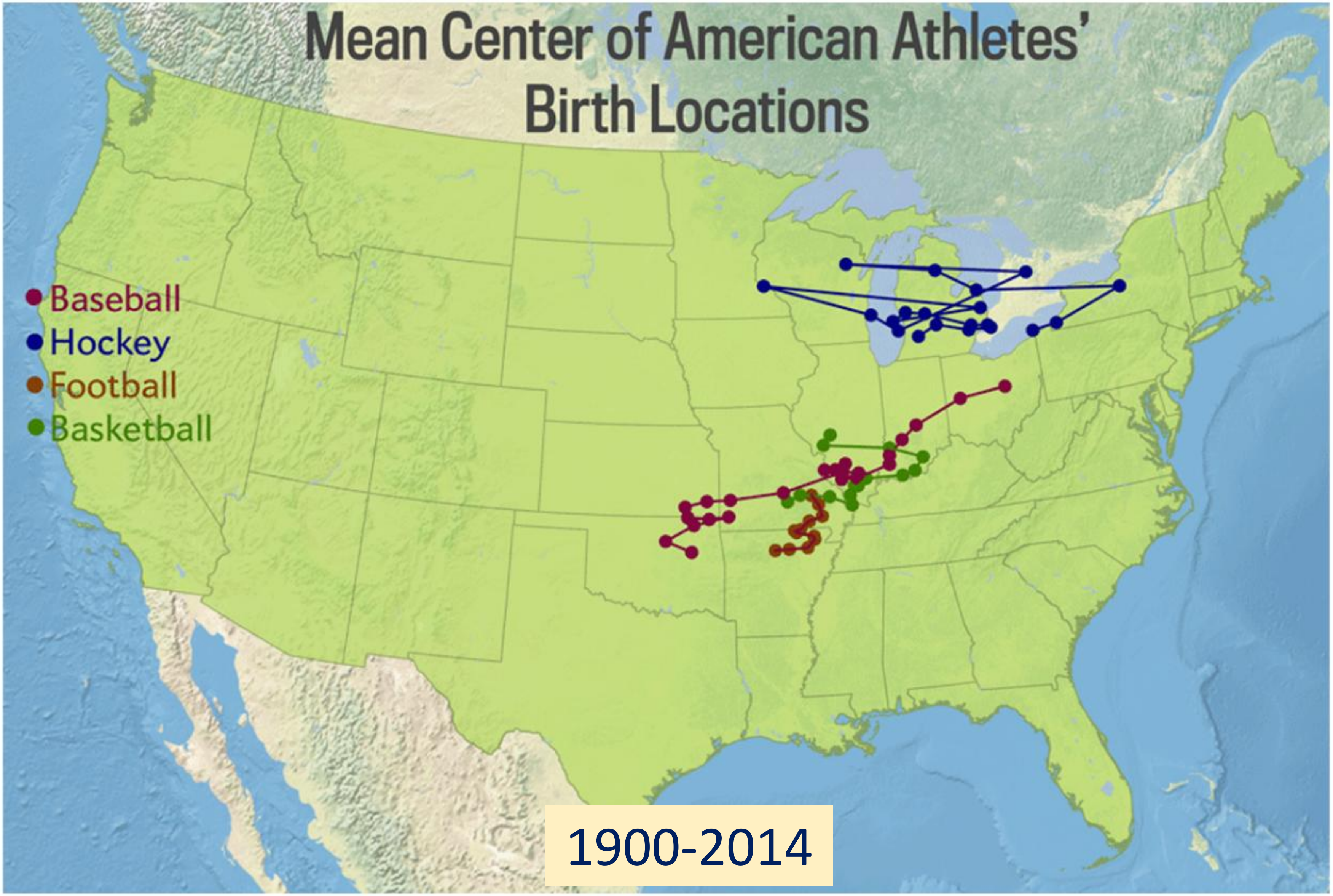
# Position of the Geographic Center of Area, Mean and Median Centers of Population: 2010


https://www2.census.gov/geo/pdfs/reference/cenpop2010/centerpop_geographic2010.pdf

# Mean centers of the United States population



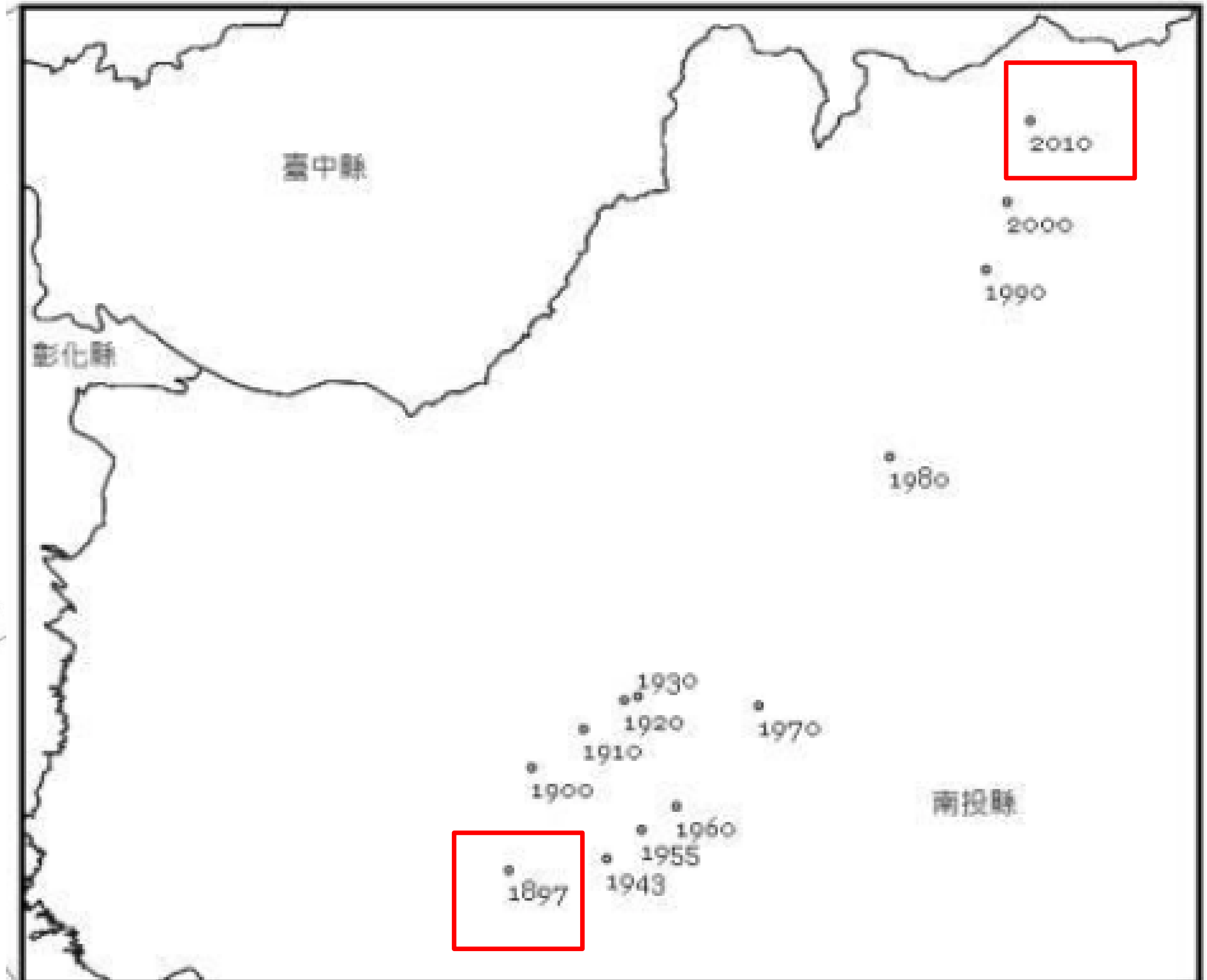

Map showing changes to the mean center of population
for the United States, 1790–2010 (US Census Bureau)




Data from sports-reference.com. Created by Ben Blatt

http://www.slate.com/articles/sports/culturebox/2014/10/baseball_player_map_a_new_u_s_map_based_on_where_baseball_players_were_born.html

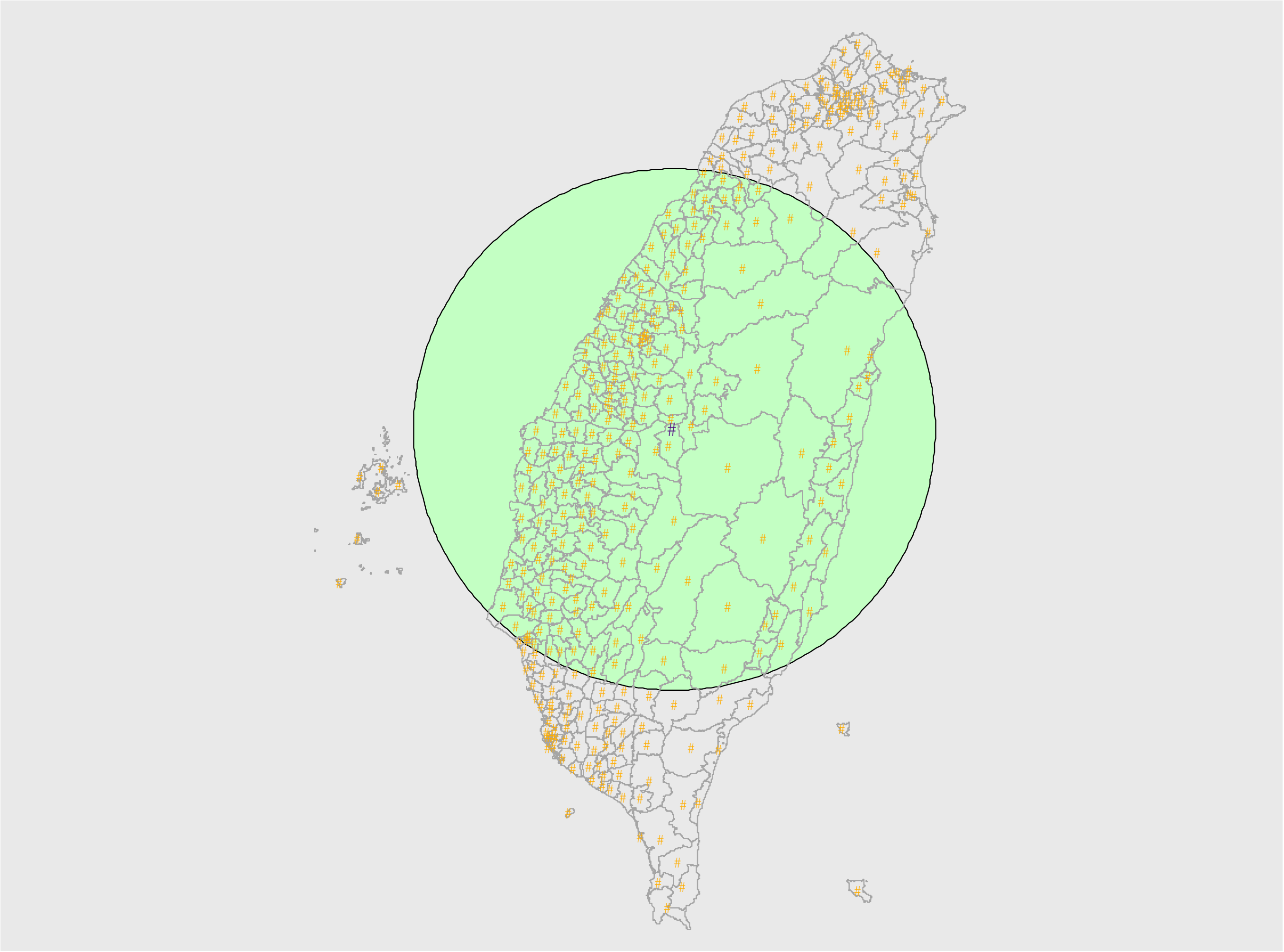
# 台灣人口中心的變遷：1897-2010



# Measures of Dispersion

- Standard Distance $$SD = \sqrt{\frac{\sum (x_i - x_{mc})^2 + \sum (y_i - y_{mc})^2}{n}}$$
  - **A spatial analogy of Std. Dev. to describe the distribution of points around the mean center**
  - **Does not capture any directional bias and capture the shape of the distribution**
- Weighted Std. Distance $$SD = \sqrt{\frac{\sum w_i (x_i - x_{mc})^2 + \sum w_i (y_i - y_{mc})^2}{\sum w_i}}$$
- Standard Deviational Ellipse
  - gives dispersion in two dimensions

# Standard Distance

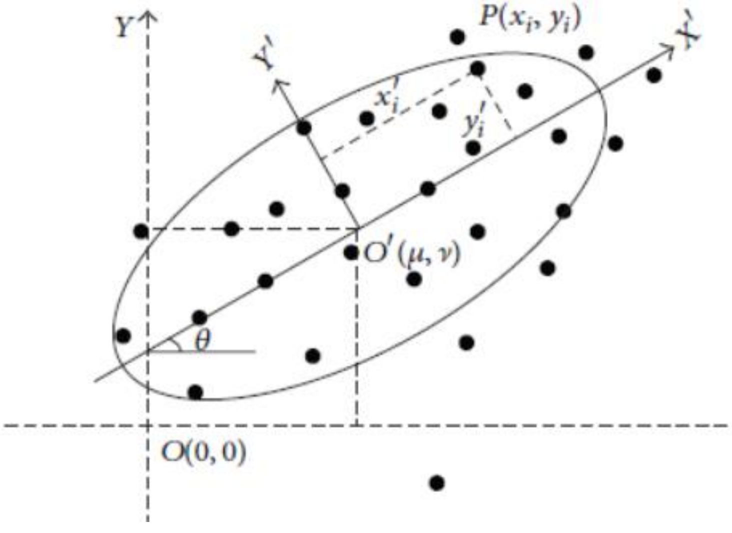


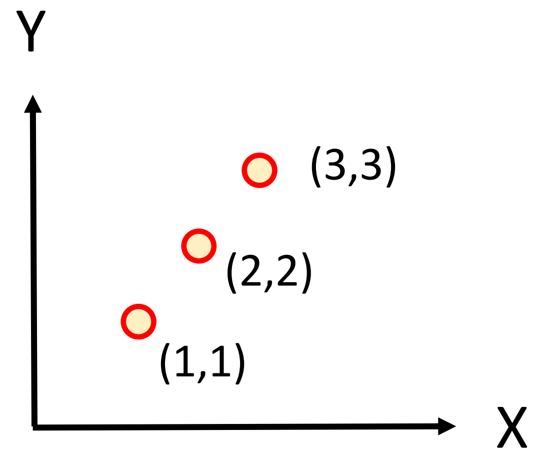
# Standard Deviational Ellipse

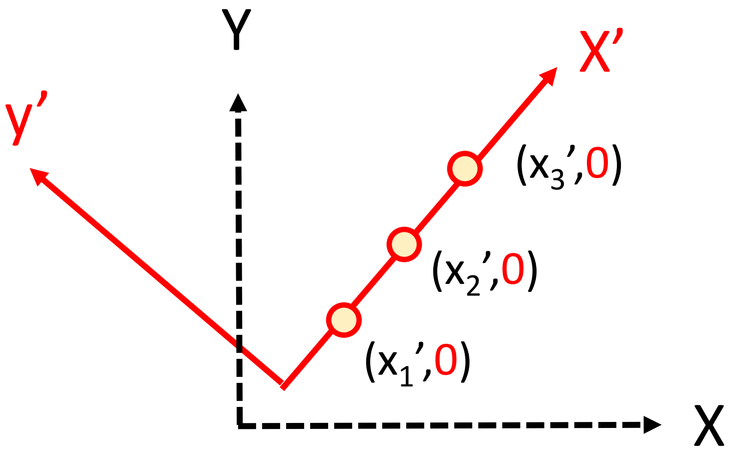
長(短)軸的長度
(標準差)

$$\sigma_x = \sqrt{2}\sqrt{\frac{\sum_{i=1}^{n}(\tilde{x}_i \cos\theta - \tilde{y}_i \sin\theta)^2}{n}}$$

$$\sigma_y = \sqrt{2}\sqrt{\frac{\sum_{i=1}^{n}(\tilde{x}_i \sin\theta + \tilde{y}_i \cos\theta)^2}{n}}$$



When the $\sigma_{y'}$ value obtains the minimum value in the rotated coordinate system, the rotated angle $\theta$ is the direction of scattered points. Then it can get the minimum value through calculating the derivative of $\sigma_{y'}$ for (12)

$$\frac{d\sigma_{y'}}{d\theta} = \frac{1}{n\sigma_{y'}}\left[\sum_{i=1}^{n}\overline{x_i}^2 \sin\theta\cos\theta - \sum_{i=1}^{n}\overline{x_i}\,\overline{y_i}\left(\cos^2\theta - \sin^2\theta\right) - \sum_{i=1}^{n}\overline{y_i}^2 \sin\theta\cos\theta\right]. \tag{14}$$

Solving for $\theta$,

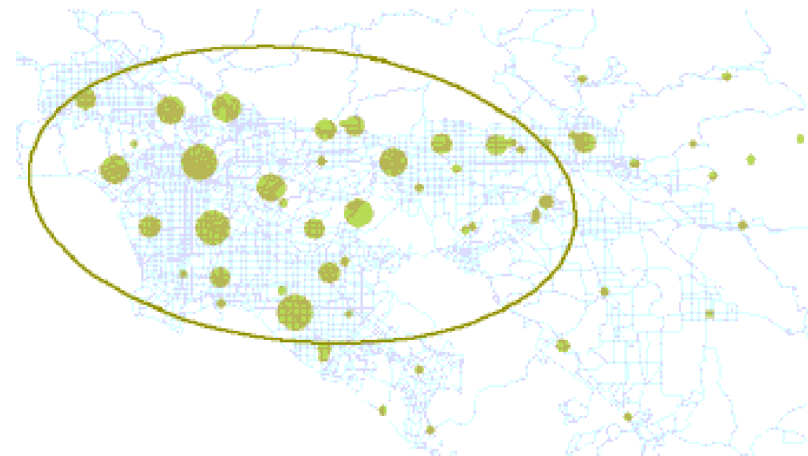
$$\tan\theta = \frac{\left(\sum_{i=1}^{n}\overline{x_i}^2 - \sum_{i=1}^{n}\overline{y_i}^2\right) \pm \sqrt{\left(\sum_{i=1}^{n}\overline{x_i}^2 - \sum_{i=1}^{n}\overline{y_i}^2\right)^2 + 4\left(\sum_{i=1}^{n}\overline{x_i}\,\overline{y_i}\right)^2}}{2\sum_{i=1}^{n}\overline{x_i}\,\overline{y_i}}. \tag{15}$$

# Standard Deviational Ellipse

X-Y軸的標準差

$$SDE_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{X})^2}{n}}$$

$$SDE_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{Y})^2}{n}}$$



旋轉角度

$$\tan\ \theta = \frac{A+B}{C}$$

$$A = \left(\sum_{i=1}^{n}\tilde{x}_i^2 - \sum_{i=1}^{n}\tilde{y}_i^2\right)$$

$$B = \sqrt{\left(\sum_{i=1}^{n}\tilde{x}_i^2 - \sum_{i=1}^{n}\tilde{y}_i^2\right)^2 + 4\left(\sum_{i=1}^{n}\tilde{x}_i\tilde{y}_i\right)^2}$$

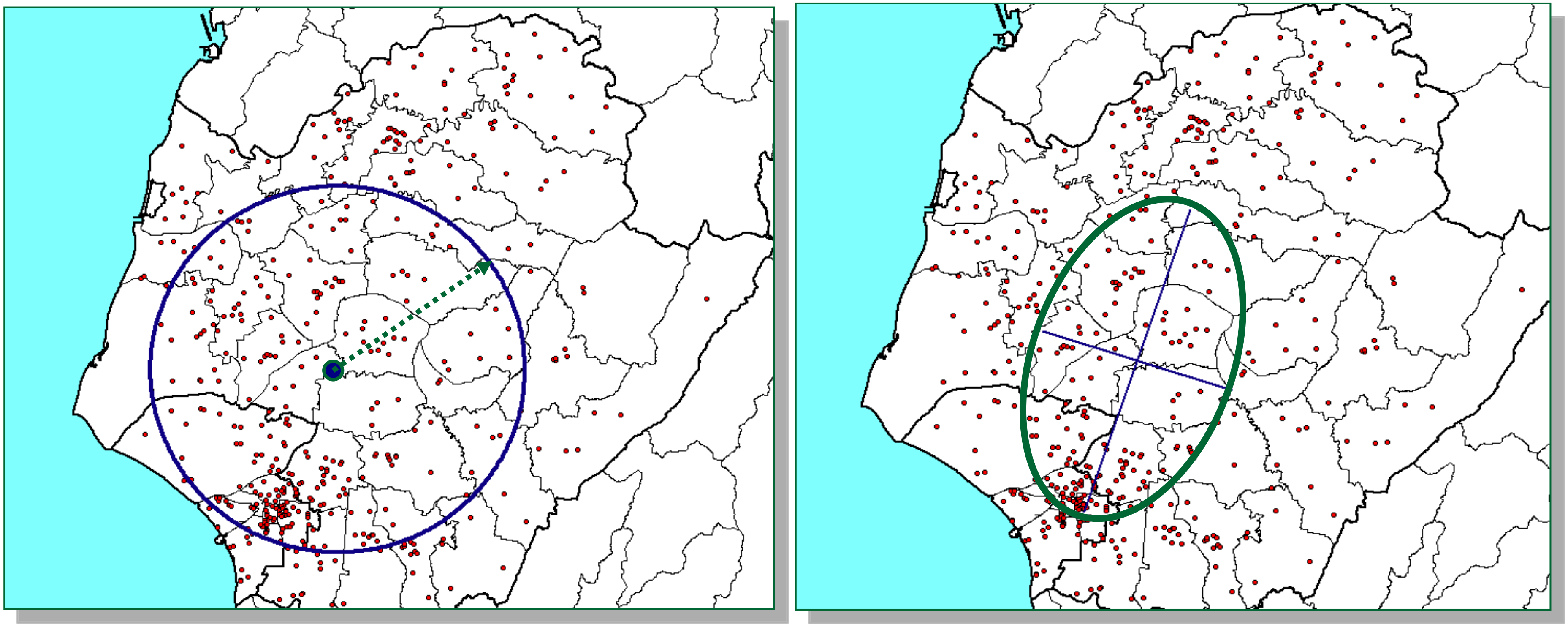
$$C = 2\sum_{i=1}^{n}\tilde{x}_i\tilde{y}_i$$

長(短)軸的長度
(標準差)

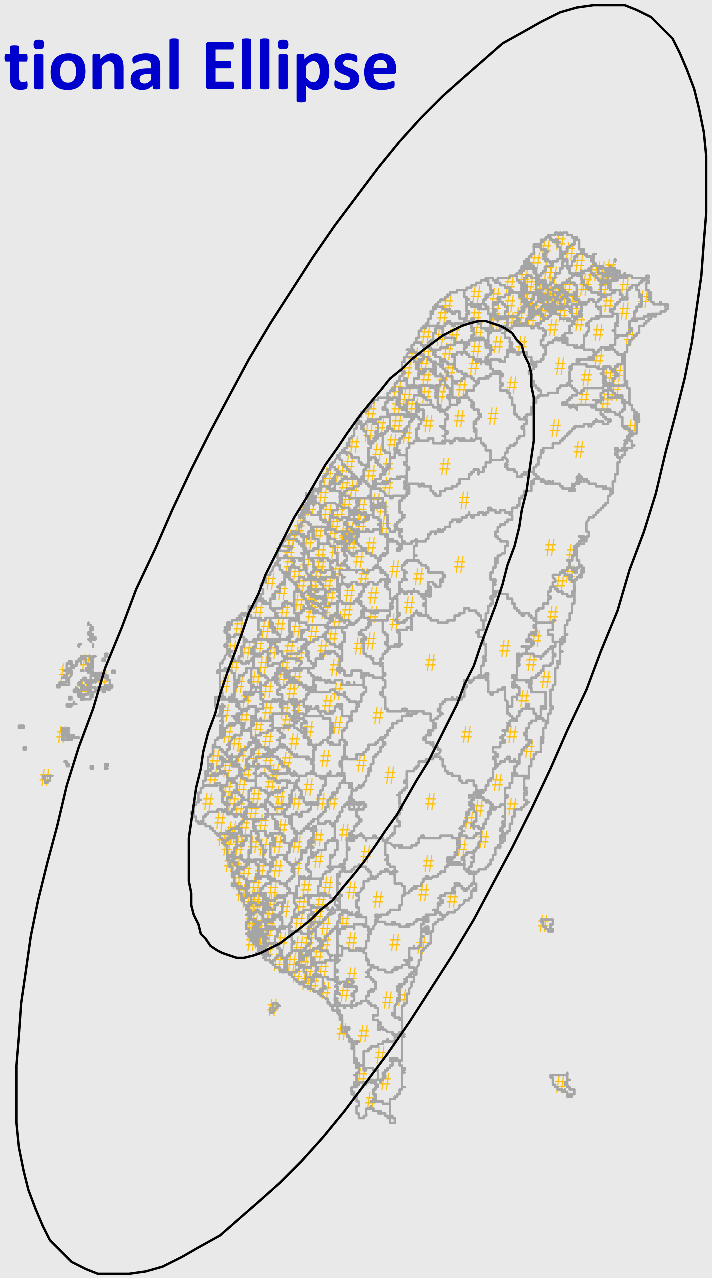
$$\sigma_x = \sqrt{2}\sqrt{\frac{\sum_{i=1}^{n}(\tilde{x}_i\ \cos\ \theta - \tilde{y}_i\ \sin\ \theta)^2}{n}}$$

$$\sigma_y = \sqrt{2}\sqrt{\frac{\sum_{i=1}^{n}(\tilde{x}_i\ \sin\ \theta + \tilde{y}_i\ \cos\ \theta)^2}{n}}$$

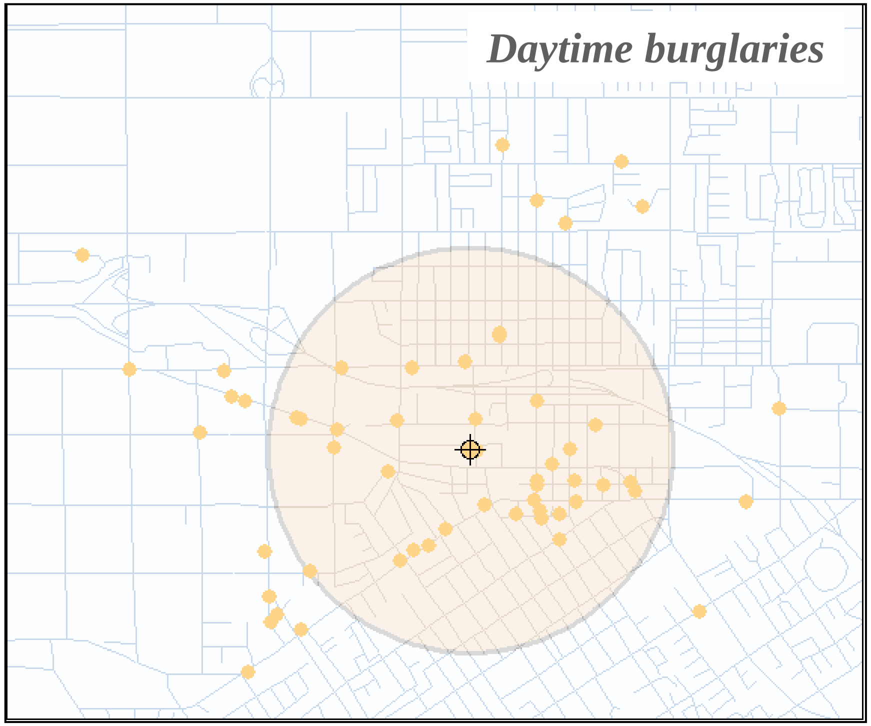
# Std Distance vs. Std Deviational Ellipse

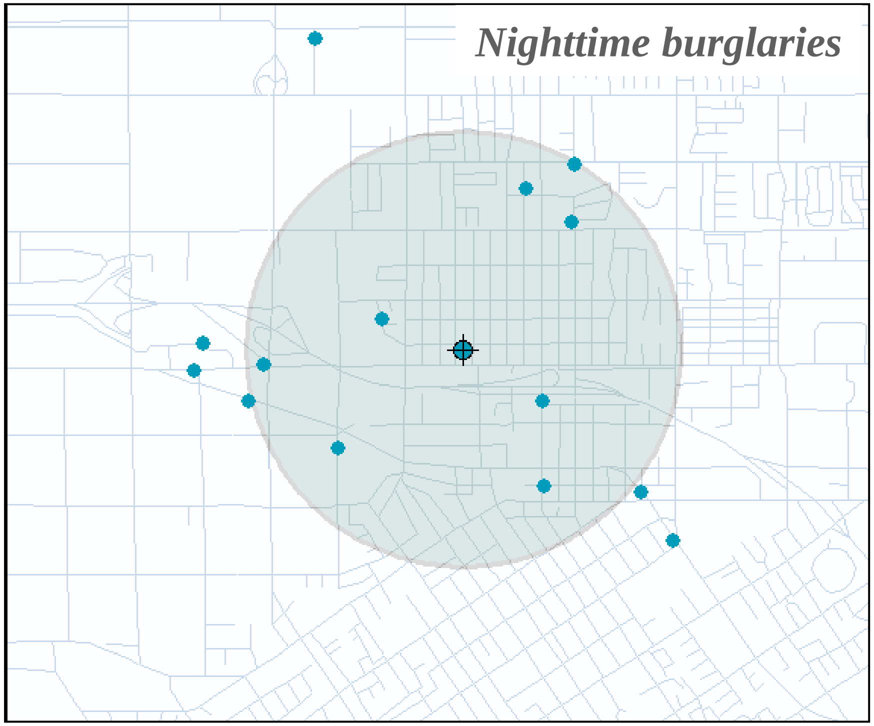


# Standard Deviational Ellipse



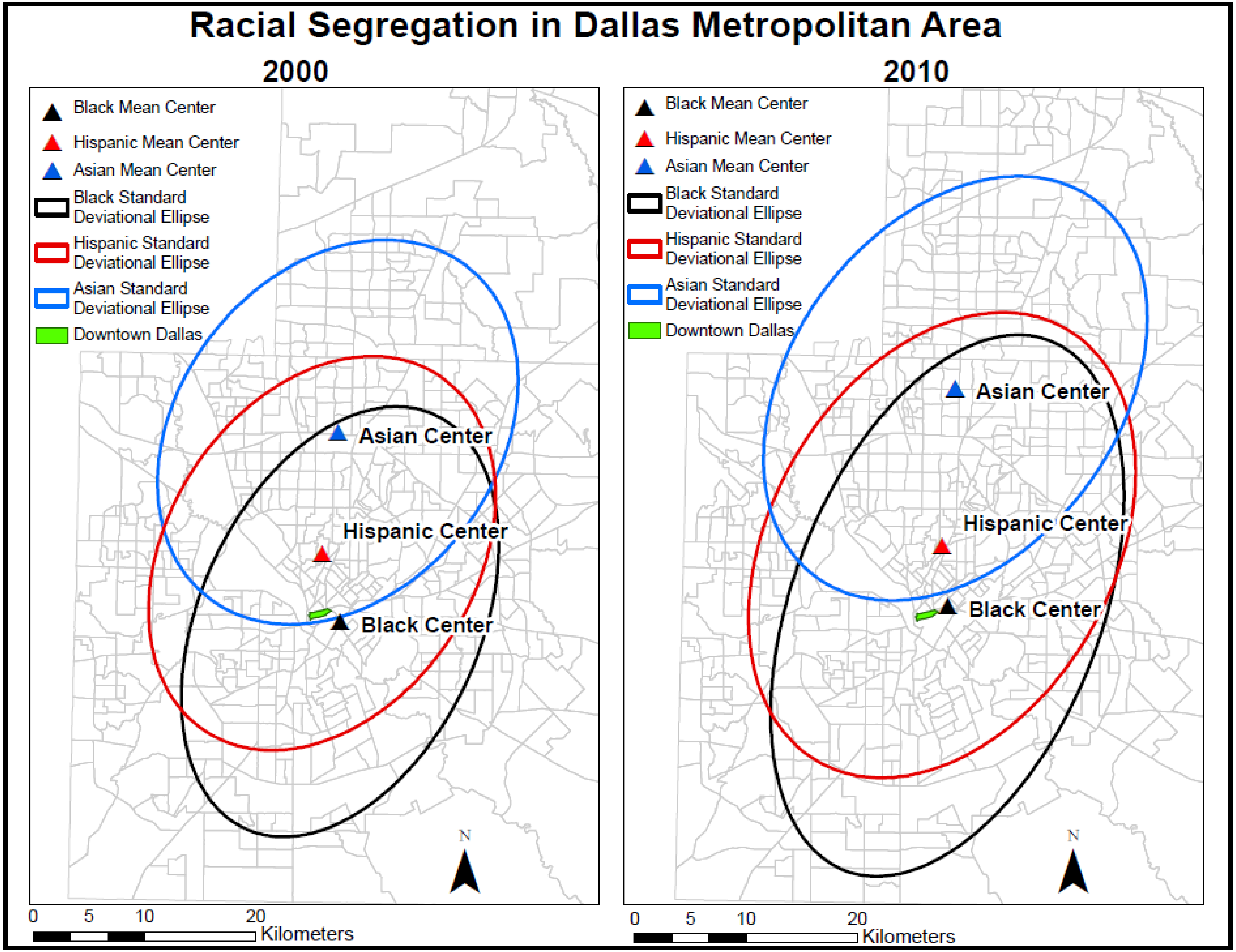
# Comparisons of spatial distributions




***1. Daytime burglaries tend to be slightly more concentrated***

***2. The differences in the locations***

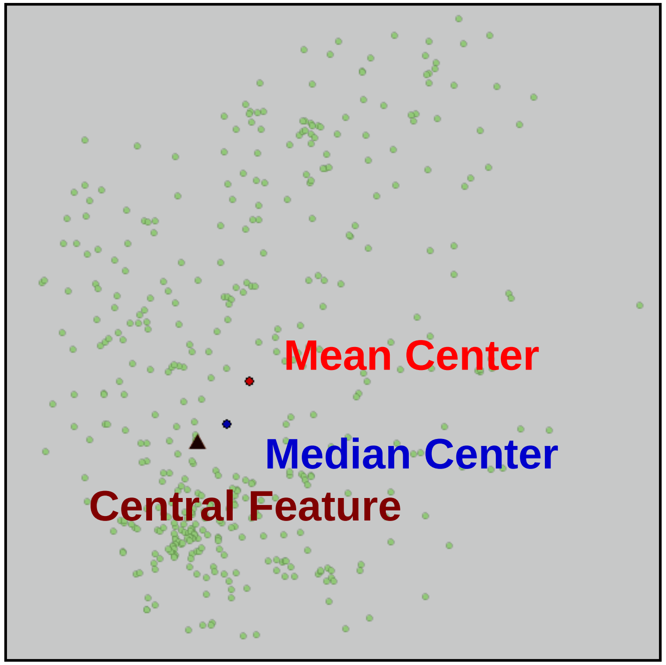


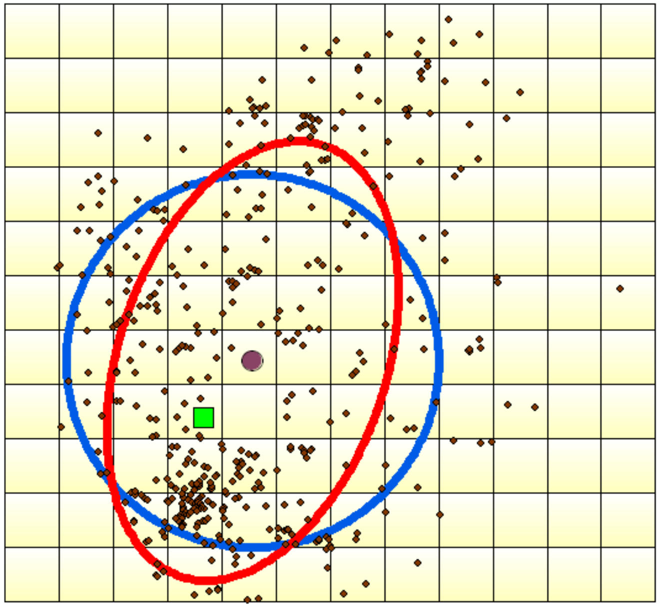

*Projection: USA Contiguous Albers Equal Area Conic*
*Data Source: US Census Tracts*

Produced by Heather Cormier on February 5, 2014

# Demo：Describing a Point Pattern

- 提供圖資(schools.shp)：
  - 台南地區學校分佈
- 分析方法：
  - Spatial Distribution
    - Spatial Mean Center
    - Median Center
    - Central Feature
    - Standard Distance
    - Standard Deviational Ellipse



# Sample Data: schools_sf

| | AREA | PERIMETER | SCHOOL_ | SCHOOL_ID | NAME | X_coor | Y_coor | ID1 | NEAR_FID | NEAR_DIST | geometry |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 38 | 39 | 土城子國小分校 | 156258.5 | 2550982 | 39 | 29 | 3293.07480 | c(156258.5, 2550982) |
| 2 | 0 | 0 | 81 | 82 | 鎮海國小 | 160475.1 | 2546815 | 82 | 18 | 668.74035 | c(160475.09375, 2546815) |
| 3 | 0 | 0 | 82 | 83 | 幼稚園 | 162749.6 | 2545007 | 83 | 26 | 327.91552 | c(162749.59375, 2545007) |
| 4 | 0 | 0 | 86 | 87 | 慈幼高工 | 171221.4 | 2543562 | 87 | 4 | 139.67050 | c(171221.40625, 2543562) |
| 5 | 0 | 0 | 87 | 88 | 母佑幼稚園 | 171159.1 | 2543437 | 88 | 3 | 139.67050 | c(171159.09375, 2543437) |
| 6 | 0 | 0 | 88 | 89 | 牧群幼稚園 | 171164.3 | 2543261 | 89 | 4 | 176.07689 | c(171164.296875, 2543261) |
| 7 | 0 | 0 | 89 | 90 | 復興國中 | 171283.6 | 2542581 | 90 | 197 | 666.11873 | c(171283.59375, 2542581) |
| 8 | 0 | 0 | 93 | 94 | 南興國小 | 163112.5 | 2552910 | 94 | 12 | 1562.81303 | c(163112.5, 2552910) |
| 9 | 0 | 0 | 98 | 99 | 瀛海高中 | 168313.2 | 2548684 | 99 | 69 | 756.48794 | c(168313.203125, 2548684) |
| 10 | 0 | 0 | 100 | 101 | 安和幼稚園 | 168512.3 | 2547675 | 101 | 69 | 673.41490 | c(168512.296875, 2547675) |
| 11 | 0 | 0 | 101 | 102 | 安順國小 | 169022.0 | 2550011 | 102 | 79 | 211.31126 | c(169022, 2550011) |
| 12 | 0 | 0 | 108 | 109 | 學甲托兒所 | 161381.1 | 2553384 | 109 | 12 | 240.38739 | c(161381.09375, 2553384) |

# 0. Data Preparation

```
# Generating no. of student in each school
schools_sf$Students<-as.integer (runif(424,100,1000))

# Generating school type: cluster vs. isolation
for (i in 1: 424) {

  if (schools_sf$NEAR_DIST[i]< 500) {
    schools_sf$type[i]<- "Cluster"
  } else schools_sf$type[i]<- "Isolation"

}

index<- schools_sf$type == "Cluster"
school_cluster <- schools_sf[index,]

length(school_cluster)
schools.c_lyr <- tm_shape(school_cluster)+tm_dots(col="red", size= 0.1) + tm_layout(frame = F)

School_df <- data.frame(x=schools_sf$X_coor, y=schools_sf$Y_coor,
                        type=schools_sf$type, students=schools_sf$Students)
```

產生連續數值的屬性

產生類別數值的屬性

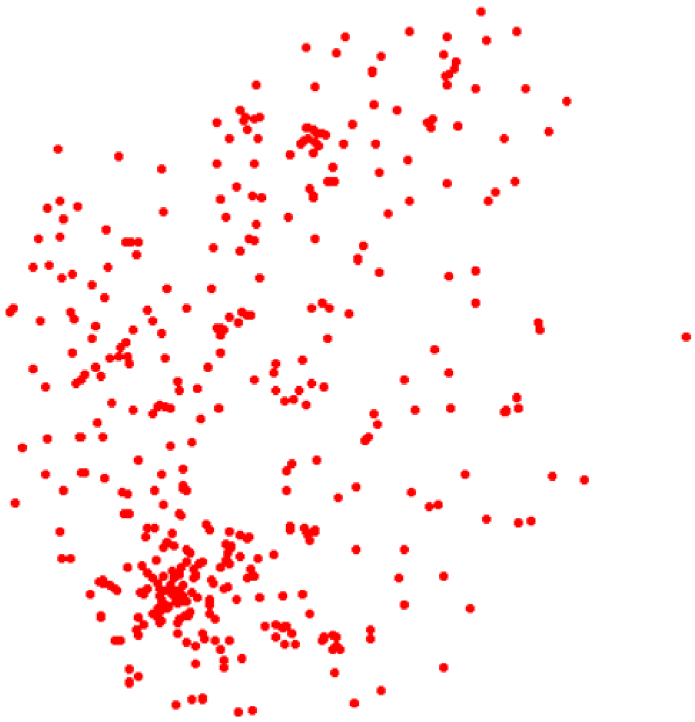
擷取特定屬性的圖資

# 0. Data Preparation: School_df

```
> School_df
           x       y      type students
1   156258.5 2550982 Isolation      506
2   160475.1 2546815 Isolation      405
3   162749.6 2545007   Cluster      661
4   171221.4 2543562   Cluster      605
5   171159.1 2543437   Cluster      960
6   171164.3 2543261   Cluster      120
7   171283.6 2542581 Isolation      898
8   163112.5 2552910 Isolation      720
9   168313.2 2548684 Isolation      473
10  168512.3 2547675 Isolation      427
11  169022.0 2550011   Cluster      459
12  161381.1 2553384   Cluster      155
```

# Comparisons

schools_sf



school_cluster

# R packages: aspace

aspace (version 4.1.0)

## aspace-package: A collection of functions for estimating centrographic statistics and computational geometries for spatial point patterns

### Description

A collection of functions for computing centrographic statistics (e.g., standard distance, standard deviation ellipse, standard deviation box) for observations taken at point locations. The 'aspace' package was originally conceived to aid in the analysis of spatial patterns of travel behaviour (see Buliung and Remmel, 2008).

https://www.rdocumentation.org/packages/aspace/versions/4.1.0/topics/aspace-package

# 1. Mean and Median Centers

```
Mean.Center <- calc_mnc(id=1, points=School_df[,1:2])

Median.Center <- calc_mdc(id=1, points=School_df[,1:2])
```

| | |
|---|---|
| Mean.Center | List of 6 |

| Name | Type | Value |
|---|---|---|
| Mean.Center | list [6] | List of length 6 |
| TYPE | character [1] | 'MNC' |
| DATE | character [1] | 'Sun Mar 24 13:34:45 2024' |
| ID | double [1] | 1 |
| LOCATIONS | double [1 x 3] | 1 173824 2557130 |
| FORPLOTTING | list [6] | List of length 6 |
| ATTRIBUTES | list [1 x 3] (S3: data.frame) | A data.frame with 1 row and 3 columns |

# 1. Mean and Median Centers

## 建立 sf 格式的point data

- `sf`, the table (`data.frame`) with feature attributes and feature geometries, which contains
- `sfc`, the list-column with the geometries for each feature (record), which is composed of
- `sfg`, the feature geometry of an individual simple feature.

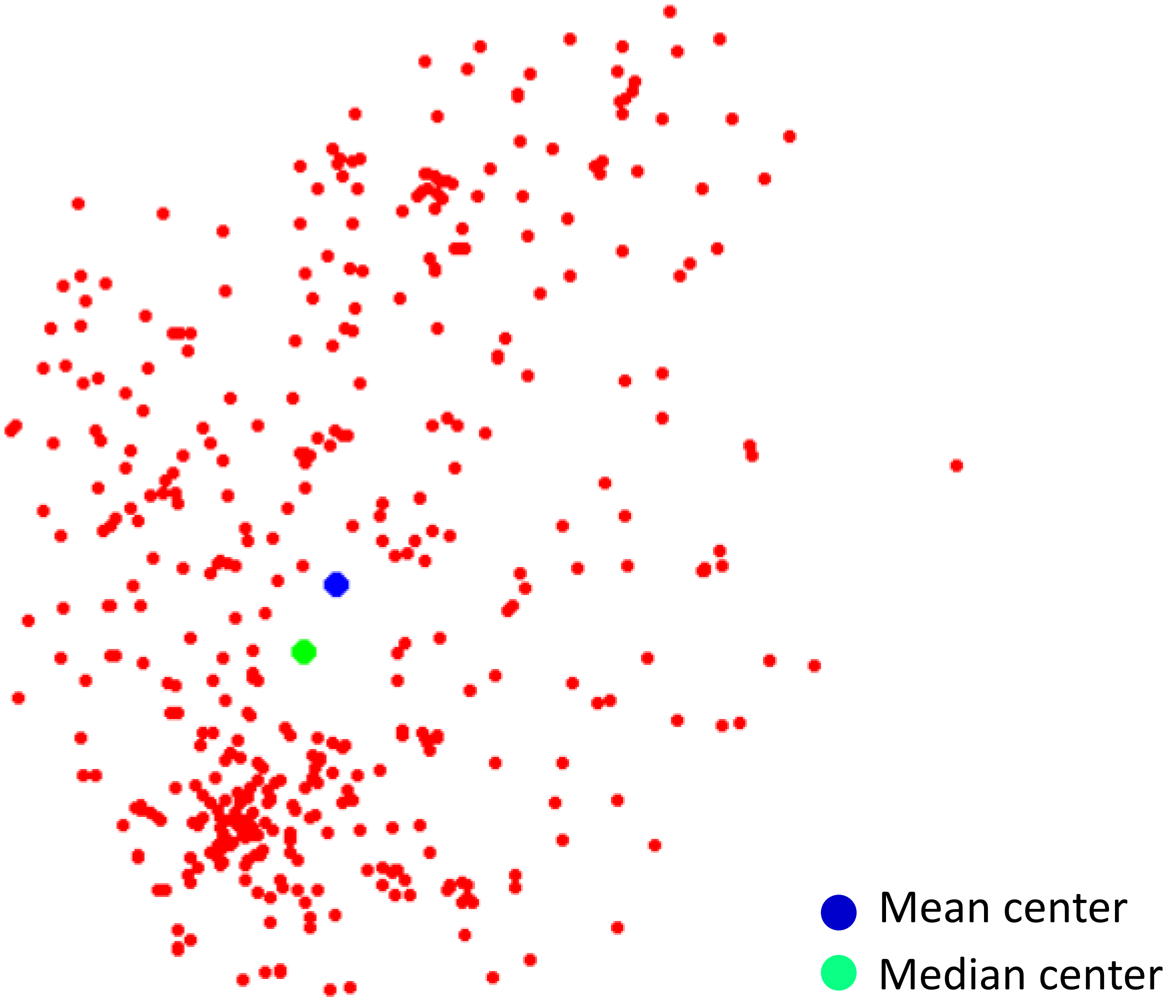
```
Median.Center_sfg = st_point(c(Mean.xcoor, Mean.ycoor))

Median.Center_sfc = st_sfc(Median.Center_sfg)

Median.Center_sf <- st_sf(Median.Center_sfc)

st_crs(Median.Center_sf) <- st_crs(schools_sf)

tm_shape(Median.Center_sf)+tm_dots(fill="green", size= 0.5)
```

# 補充： %>% 運算元

```
Mean.Center_sfg = st_point(c(Mean.Center[,2],Mean.Center[,3]))
Mean.Center_sfc = st_sfc(Mean.Center_sfg)
Mean.Center_sf <- st_sf(Mean.Center_sfc)
```

```
Mean.Center.Coor <- c(Mean.xcoor, Mean.ycoor)
Mean.Center_sf <- Mean.Center.Coor %>% st_point %>% st_sfc %>% st_sf
```

# Comparisons



# 2. Weighted Mean Center

```
W.Mean.Center <- calc_mnc(id=1, weighted=TRUE, weights=School_df$students,
                          points=School_df[,1:2])

W.Mean.xcoor <- W.Mean.Center$LOCATIONS[2]
W.Mean.ycoor <- W.Mean.Center$LOCATIONS[3]
W.Mean.Center.Coor <- c(W.Mean.xcoor, W.Mean.ycoor)

W.Mean.Center_sf <- W.Mean.Center.Coor %>% st_point %>% st_sfc %>% st_sf
st_crs(W.Mean.Center_sf) <- st_crs(schools_sf)
tm_shape(W.Mean.Center_sf)+tm_dots(fill="green", size= 0.5)
```

# 3. SDD: Standard Distance

school.SDD<- calc_sdd (id=1, points=School_df[,1:2])

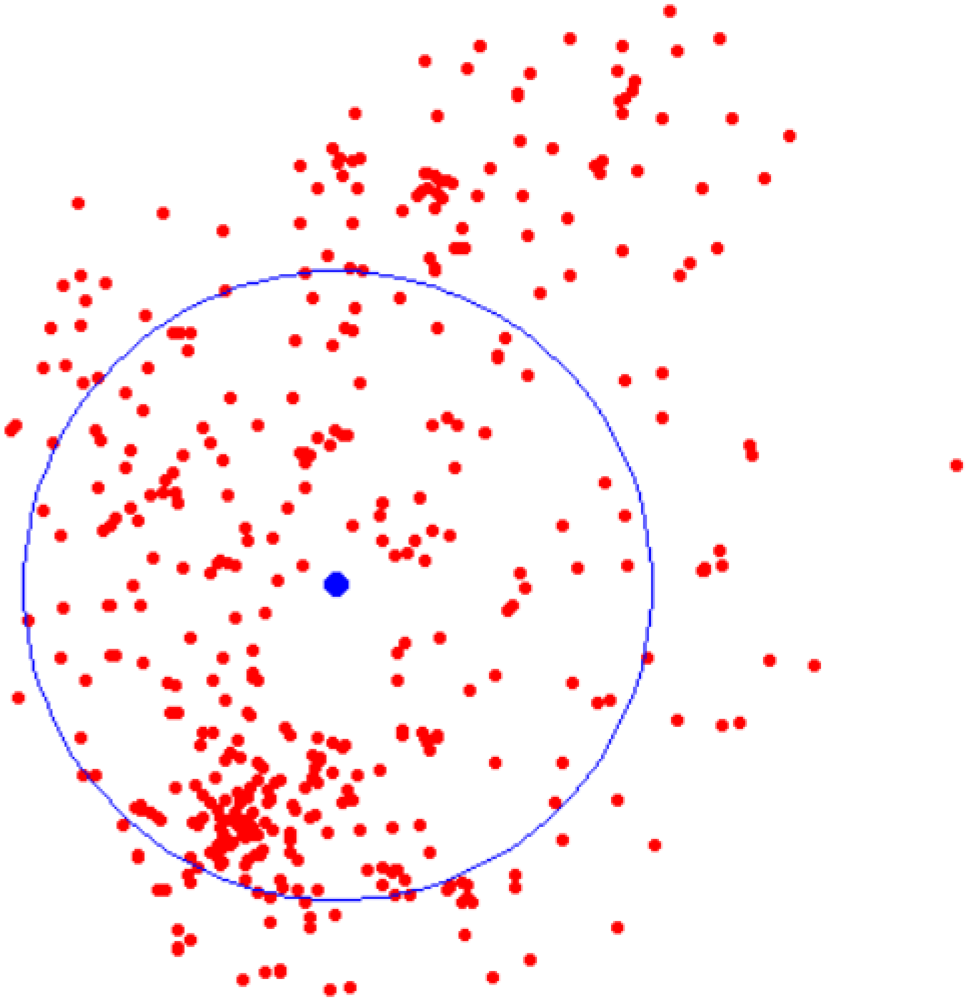
school.SDD List of 6

| Name | Type | Value |
|---|---|---|
| school.SDD | list [6] | List of length 6 |
| TYPE | character [1] | 'SDD' |
| DATE | character [1] | 'Sun Mar 24 14:05:26 2024' |
| ID | double [1] | 1 |
| LOCATIONS | list [360 x 3] (S3: data.frame) | A data.frame with 360 rows and 3 columns |
| FORPLOTTING | list [10] | List of length 10 |
| ATTRIBUTES | list [1 x 7] (S3: data.frame) | A data.frame with 1 row and 7 columns |

# Mapping Standard Distance: R code

```
center.x<- school.SDD$ATTRIBUTES$CENTRE.x
center.y<- school.SDD$ATTRIBUTES$CENTRE.y
center.coor <- c(center.x, center.y)

center_sf<- center.coor %>% st_point %>% st_sfc %>% st_sf
st_crs(center_sf) <-  st_crs(schools_sf)

rad<- school.SDD$ATTRIBUTES$SDD.radius
SD_sf<- st_buffer(center_sf, rad)
SD_lyr <- tm_shape(SD_sf) + tm_borders(col = "blue") +
            tm_layout(frame = F)
Center_lyr <-  tm_shape(center_sf) + tm_dots(fill="blue", size= 0.5)

schools_lyr+ Center_lyr+ SD_lyr
```

# Mapping Standard Distance（results）



# Weighted Standard Distance

```
school.SDD2<- calc_sdd(id=1, points = School_df[,1:2],
                       weighted = TRUE, weights=School_df$students)
```

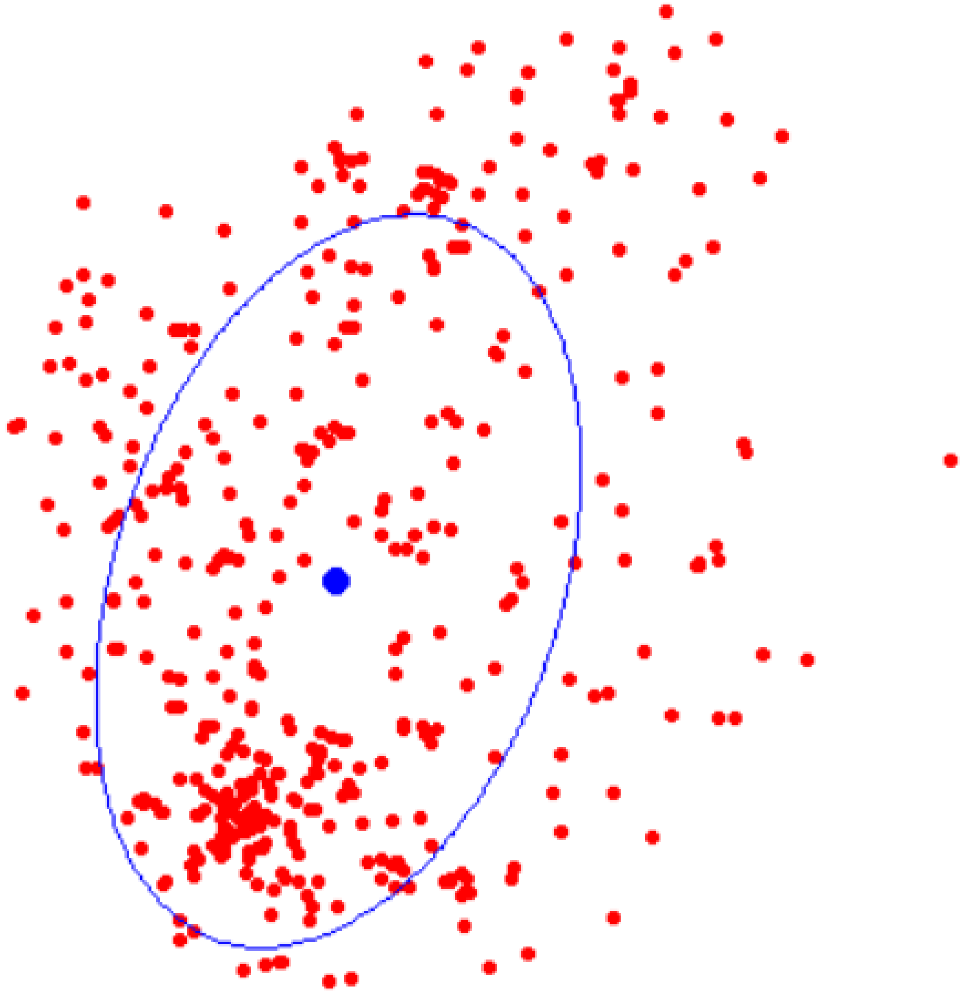
# 4. SDE: Standard Deviational Ellipse

```
school.SDE <- calc_sde(id=1, points=School_df[,1:2])
```

| | |
|---|---|
| school.SDE | List of 6 |

| Name | Type | Value |
|---|---|---|
| school.SDE | list [6] | List of length 6 |
| TYPE | character [1] | 'SDE' |
| DATE | character [1] | 'Sun Mar 24 14:09:04 2024' |
| ID | double [1] | 1 |
| LOCATIONS | list [360 x 3] (S3: data.frame) | A data.frame with 360 rows and 3 columns |
| FORPLOTTING | list [22] | List of length 22 |
| ATTRIBUTES | list [1 x 19] (S3: data.frame) | A data.frame with 1 row and 19 columns |

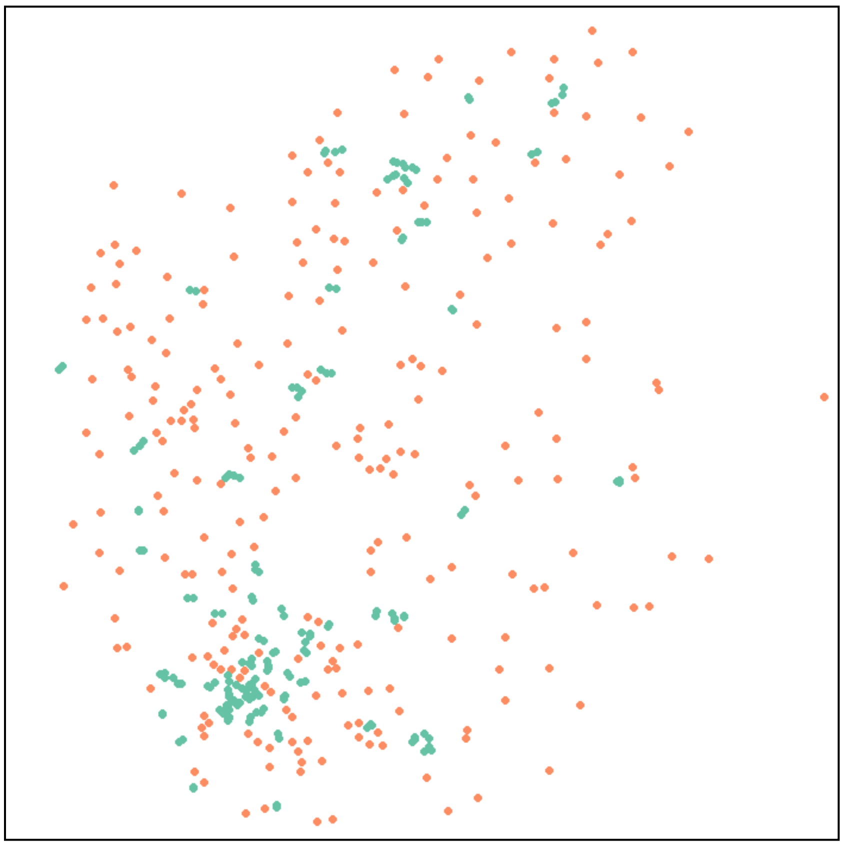
# Mapping Standard Deviational Ellipse

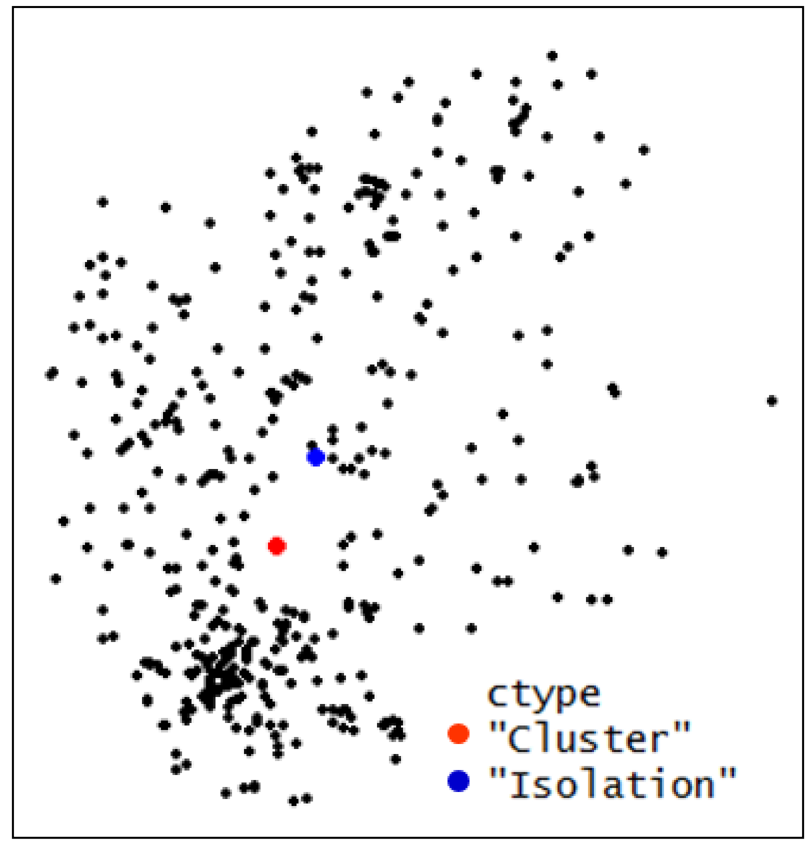


# 5. Central Feature

```
school.CF <- calc_cf(id=1, points=School_df[,1:2])
CF.x<- school.CF$ATTRIBUTES$CF.x
CF.y<- school.CF$ATTRIBUTES$CF.y
CF.coor <- c(CF.x, CF.y)

CF_sf <- CF.coor %>% st_point %>% st_sfc %>% st_sf
st_crs(CF_sf) <-  st_crs(schools_sf)

CF_lyr <- tm_shape(CF_sf) + tm_dots(fill = “blue”, size= 0.5) +
            tm_layout(frame = F)

schools_lyr + CF_lyr
```

# 6. Mean centers by grouping attributes



# Mean centers by grouping attributes

```
type <- schools_sf$type
newid<- unique(type)

xx<-vector(); yy<-vector(); ctype<-vector()
for (i in 1:2){
  index<-(type == newid[i])
  newschool<-schools_sf[index, ]
  xcoor<-newschool$X_coor
  ycoor<-newschool$Y_coor
  newschool.mc <- mean_centre(id=1, points=cbind(xcoor, ycoor))
  xx[i]<-newschool.mc$LOCATIONS[2]
  yy[i]<-newschool.mc$LOCATIONS[3]
  ctype[i]<-newid[i]
}
```

# Mean centers by grouping attributes (cont'd)

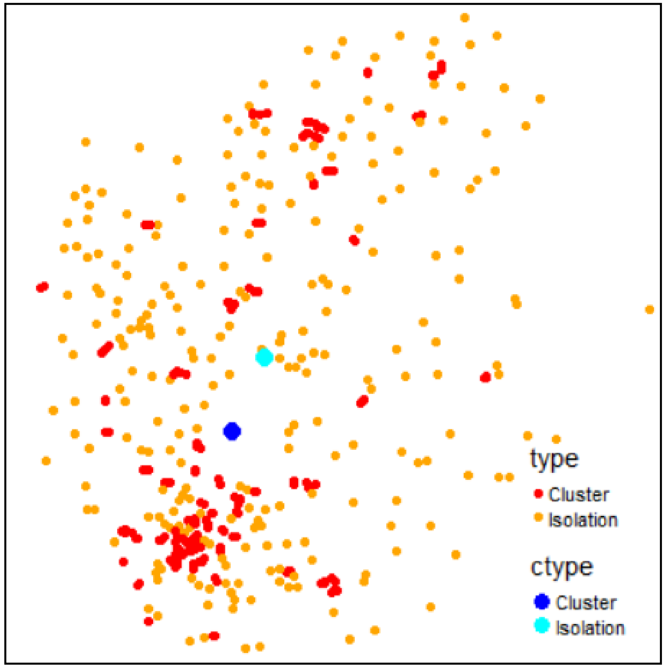
```
newcenterxy <- data.frame(xx,yy, ctype)
New_sf <- st_as_sf(newcenterxy , coords=c("xx","yy"))
st_crs(New_sf) <-  st_crs(schools_sf)
```

```
> newcenterxy
        xx      yy     ctype
1 174955.9 2559734 Isolation
2 172181.3 2553352   Cluster
```

```
> New_sf
Simple feature collection with 2 features and 1 field
Geometry type: POINT
Dimension:     XY
Bounding box:  xmin: 172181.3 ymin: 2553352 xmax: 174955.9 ymax: 2559734
Projected CRS: Transverse Mercator
      ctype                 geometry
1 Isolation POINT (174955.9 2559734)
2   Cluster POINT (172181.3 2553352)
```

# Mean centers by grouping attributes (cont'd)

```
New_lyr <- tm_shape(New_sf) +
tm_dots(col="ctype", palette=c(Cluster='blue', Isolation='cyan'), size= 0.5)
+ tm_layout(frame = F)

schools_lyr2 <- tm_shape(schools_sf) +
tm_dots(col="type", palette=c(Cluster='red', Isolation='orange'), size= 0.1)
+ tm_layout(frame = F)

schools_lyr2 + New_lyr
```



# 實習教材

https://wenlab501.github.io/GEOG2017/

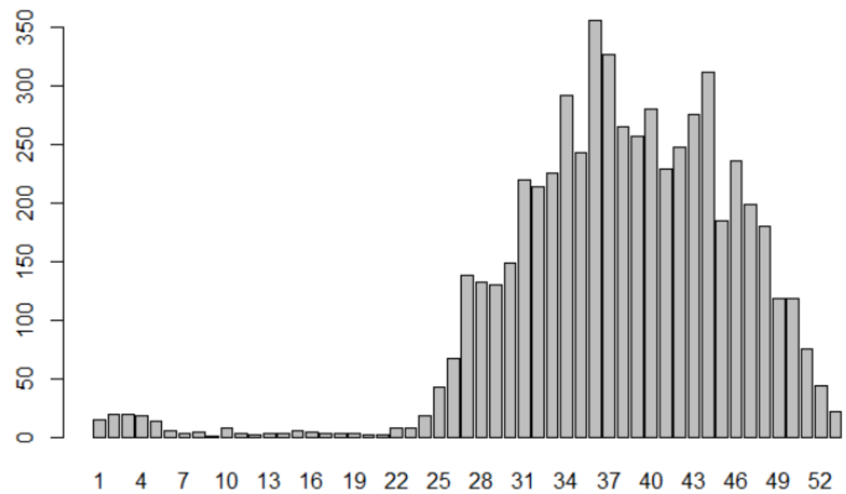
# 實習：描述疾病擴散的時空趨勢

- 圖資 **point_event.shp**
  - 登革熱病例分佈(僅為實習為目的，不是真實資料)

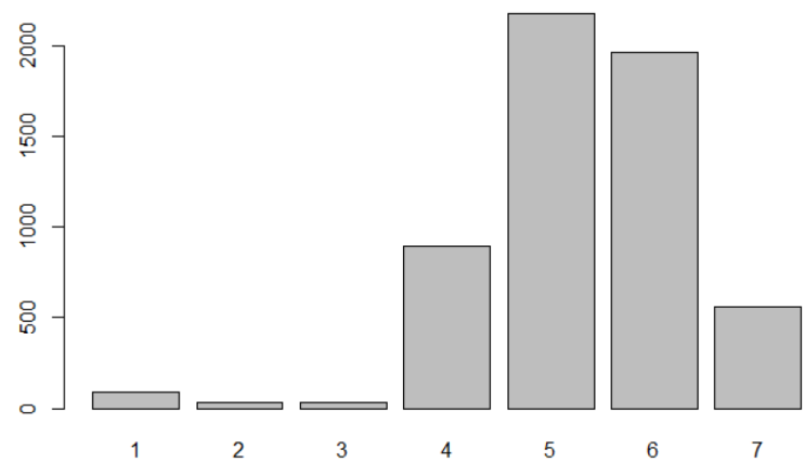
```
> Point.Attr
   WEEK      X       Y AGE KEYID coords.x1 coords.x2 optional
1    23 328891 2730820  50     1    328891   2730820     TRUE
2     3 325963 2738218  59     2    325963   2738218     TRUE
3    34 320039 2775558  26     3    320039   2775558     TRUE
4    42 313465 2611430  71     4    313465   2611430     TRUE
5    22 312964 2772731  57     5    312964   2772731     TRUE
6     4 312207 2774350  39     6    312207   2774350     TRUE
7    35 311334 2658377  32     7    311334   2658377     TRUE
8    29 310825 2775259  41     8    310825   2775259     TRUE
9    42 309045 2771607  30     9    309045   2771607     TRUE
10   40 308939 2607900  13    10    308939   2607900     TRUE
```

# 實習1: Exploring **temporal trends** in different time-scales

By week



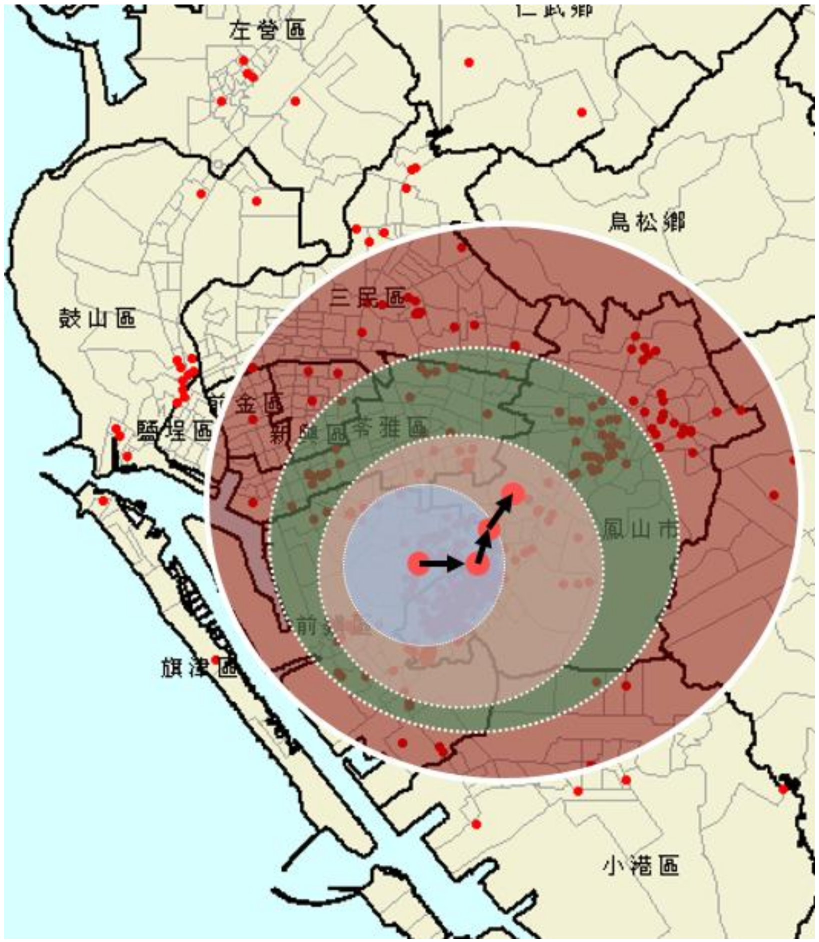

By period



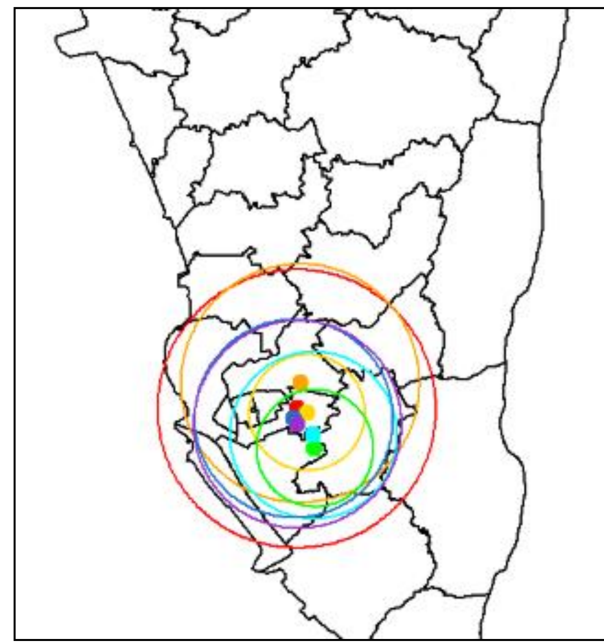

week 01~07: period 1
week 08~15: period 2
week 16~23: period 3
week 24~31: period 4
week 32~39: period 5
week 40~47: period 6
week 48~55: period 7

## 實習2 : Exploring **spatial trends** in different periods

擷取高雄地區的登革熱病例分佈




參考結果

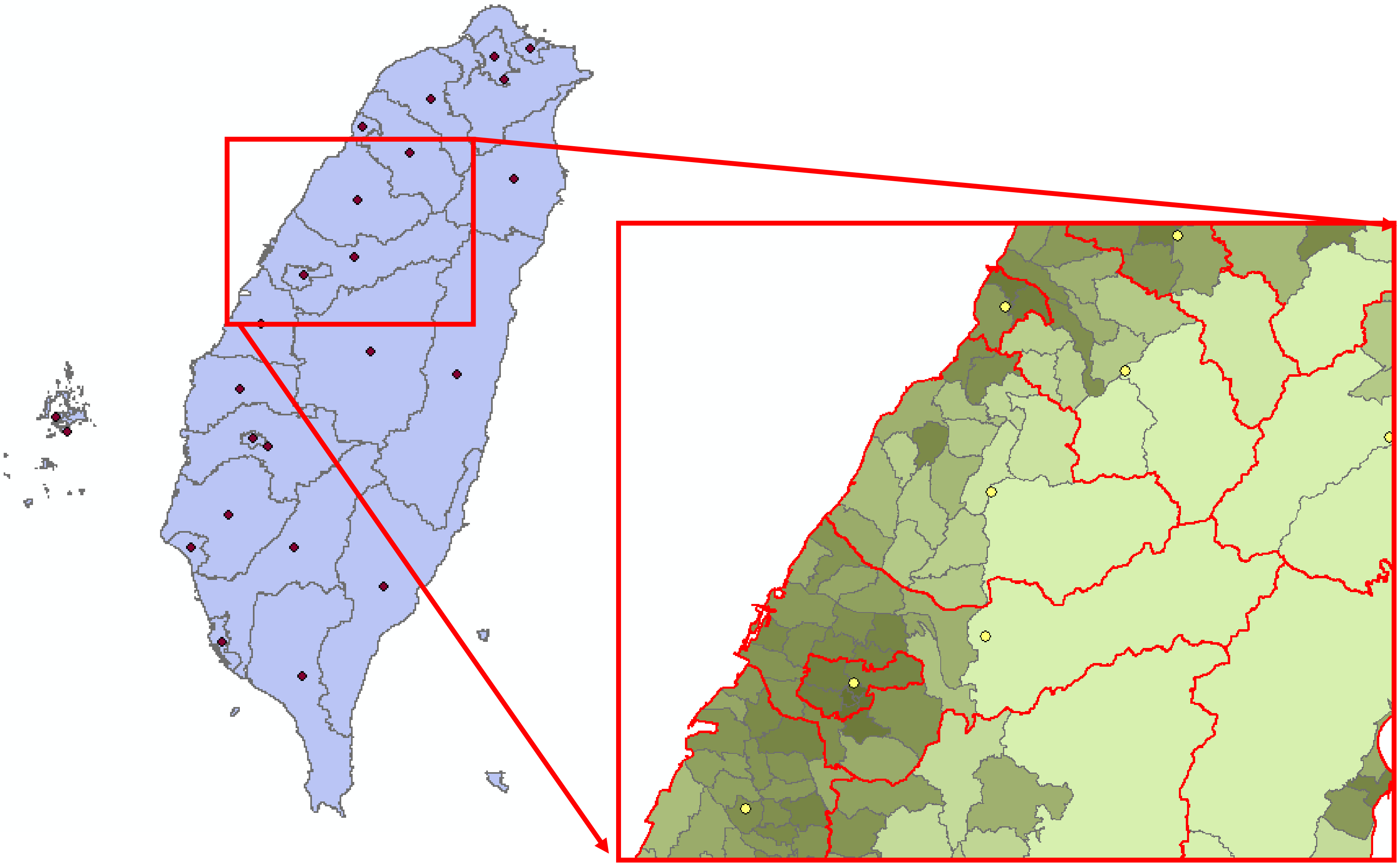


# 本週作業：計算各縣市人口中心點

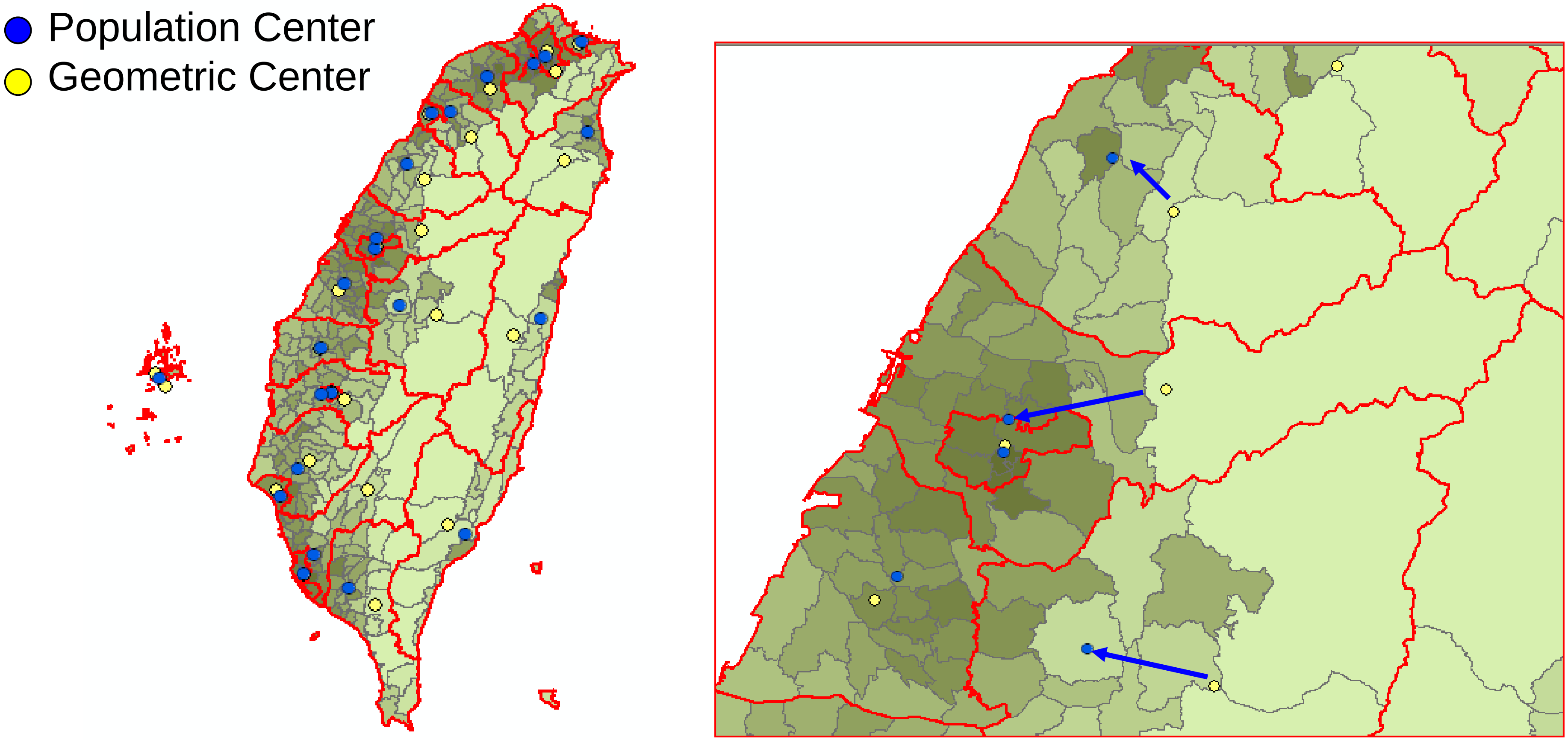
繳交截止：4/14, 下午2:00

- 圖資：
  - 台灣鄉鎮人口 Popn_TWN2.shp
- 範例內容:
  - 產生台灣縣市邊界，並計算各縣市幾何中心
  - 利用台灣鄉鎮人口加權方式，產生各縣市的人口中心點
  - 比較幾何中心點與人口中心點的差異

# 產生台灣各縣市的幾何中心點



# 利用各鄉鎮人口加權，產生各縣市的人口中心點



# 預期結果：比較各縣市的人口中心與幾何中心