熱區分析 Hot spot analysis (Localized spatial analysis)

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熱區分析的空間統計方法：第一部分

- Host-spot analysis (for polygon data)
  - Local Moran’s I index
  - Local G-statistic (Gi*)
- Issues of multiple testing for hot-spot analysis
  - Bonferroni correction
  - False discovery rate (FDR)

Textbook Chapter

TEXT_Local.Stat.pdf

Chapter 8

Local Statistics

CHAPTER OBJECTIVES

In this chapter, we:

- Explain the concepts underlying the emerging array of *local statistics*
- Account for the relatively late arrival of local statistics on the spatial analytic scene
- Review the various approaches that can be used to construct *localities* for the development of local statistics
- Discuss how the popular Getis-Ord family of G statistics are calculated and interpreted
- Outline the local version of Moran's I statistic
- Explain why inference based on local statistics is challenging and describe current approaches to dealing with the difficulties
- Provide an overview of the increasingly popular method *geographically weighted regression*
- Explain how many other spatial analysis methods can be considered as local statistics even if this was not the intent behind their original development

Chap 8: Local Statistics

p.215 - p.223

8.1 Introduction: Think geographically, measure locally

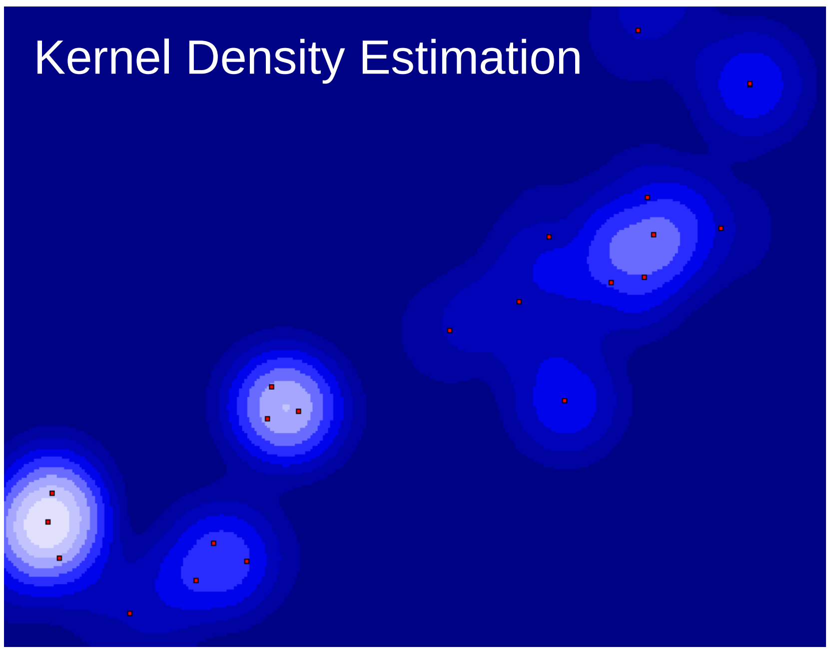
8.2 Defining the local

8.3 An example

8.4 Inference with local statistics

Identifying hot spots

Point data



Polygon data

Recap: Global Moran's I

相關係數 $$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

自相關係數
Moran's I $$I = \frac{N}{W} \frac{\sum_i \sum_j w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$

where

- N is the number of cases

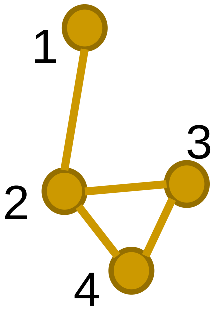
  X is the mean of the variable

  X_i is the variable value at a particular location

  X_j is the variable value at another location

  W_{ij} is a weight indexing location of i relative to j

- Applied to a continuous variable for polygons or points



	#1	#2	#3	#4
#1	0	1	0	0
#2	1	0	1	1
#3	0	1	0	1
#4	0	1	1	0

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}(y_i-\bar{y})(y_j-\bar{y}) / \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}}{\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2/n}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2/n}}$$

$$\frac{\left[\begin{aligned} &(x1-\bar{x})(x2-\bar{x}) + \\ &(x2-\bar{x})(x1-\bar{x}) + (x2-\bar{x})(x3-\bar{x}) + (x2-\bar{x})(x4-\bar{x}) + \\ &(x3-\bar{x})(x2-\bar{x}) + (x3-\bar{x})(x4-\bar{x}) + \\ &(x4-\bar{x})(x2-\bar{x}) + (x4-\bar{x})(x3-\bar{x}) \end{aligned} \right] / 8}{\dfrac{(x1-\bar{x})^2+(x2-\bar{x})^2+(x3-\bar{x})^2+(x4-\bar{x})^2}{4}}$$

Using row-standardized spatial weights

$$I = \frac{N}{W} \frac{\sum_i \sum_j w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$

$$I = \frac{\sum_i \sum_j w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$

$$I = \frac{\sum_i \sum_j \frac{w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{s_i \times s_j}}{\sum_i \frac{(x_i - \bar{x})^2}{s_i \times s_i}}$$

轉換原始分數成為 Z 分數的公式為：

$$Z = \frac{X_i - \overline{X}}{S}$$

$$\Downarrow$$

$$I = \frac{\sum_i \sum_j w_{ij} z_i z_j}{\sum_i z_i^2}$$

1. Local Moran's I
(Local Indicator of Spatial Association, LISA)

$$I = \frac{\sum_i \boxed{\sum_j w_{ij} z_i z_j}}{\sum_i z_i^2}$$

⬇

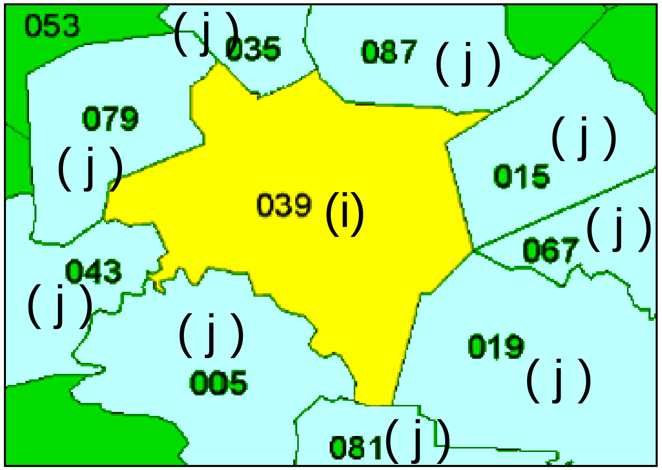
$$I_i = \frac{\sum_j w_{ij} z_i z_j}{\boxed{\sum_i z_i^2}}$$

a constant for each i

Local Moran's I (LISA)

$$I_i = z_i \sum_j w_{ij} z_j$$

$$z_i = (x_i - \bar{x}) / \delta$$



- High LISA value
  - Cluster of similar values (can be high or low)
- Low LISA value
  - Cluster of dissimilar values

Test of Statistical Significance

The z_{I_i}-score for the statistics are computed as:

$$z_{I_i} = \frac{I_i - \mathrm{E}[I_i]}{\sqrt{\mathrm{V}[I_i]}} \tag{3}$$

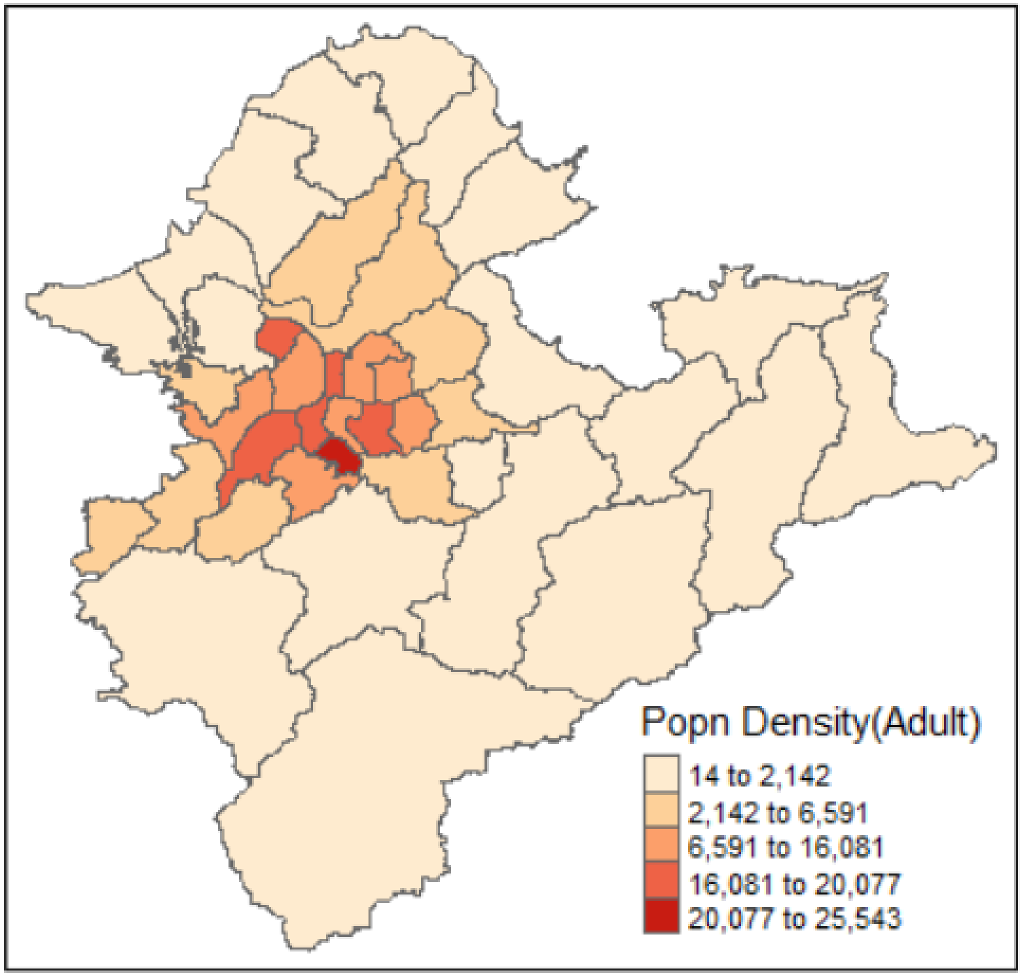
where:

$$\mathrm{E}[I_i] = -\frac{\sum_{j=1, j \neq i}^{n} w_{ij}}{n-1} \tag{4}$$

$$\mathrm{V}[I_i] = \mathrm{E}[I_i^2] - \mathrm{E}[I_i]^2 \tag{5}$$

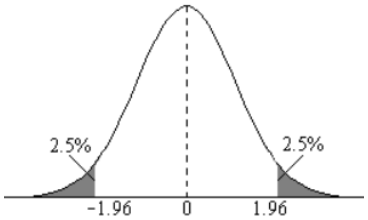
Lab: Local Moran's I

年齡15-64的人口密度 (/km^2)



Lab: Local Moran's I in R

using localmoran() function




```
localmoran(x, listw, zero.policy=NULL, na.action=na.fail, conditional=TRUE,
  alternative = "two.sided", mlvar=TRUE,
      spChk=NULL, adjust.x=FALSE)
```

greater; less; two.sided

```
> LISA.Popn <- localmoran(Density, TWN_nb_w, zero.policy=T, conditional = TRUE)
> LISA.Popn
```

	Ii	E.Ii	Var.Ii	Z.Ii	Pr(z != E(Ii))
221	0.668654386	-3.779349e-02	2.676102e-01	1.36561673	1.720593e-01
222	0.552267439	-2.314840e-02	2.139495e-01	1.24401656	2.134935e-01
223	1.122969659	-5.543082e-02	3.853030e-01	1.89841748	5.764111e-02
224	0.520540764	-1.030478e-02	6.075569e-02	2.15364779	3.126780e-02
225	1.283663684	-2.499016e-02	1.451527e-01	3.43488554	5.928042e-04
226	1.275561091	-6.572105e-02	4.518543e-01	1.99535859	4.600378e-02
227	2.310784759	-5.639251e-02	3.170003e-01	4.20437007	2.618103e-05

```
LISA.Popn <- localmoran(Density, TWN_nb_w, zero.policy=T)

NorthTW_sf$z.li <- LISA.Popn[,4]
NorthTW_sf$pvalue <- LISA.Popn[,5]
```

```
localmoran(x, listw, zero.policy=NULL, na.action=na.fail, conditional=TRUE,
    alternative = "two.sided", mlvar=TRUE,
        spChk=NULL, adjust.x=FALSE)
```

`conditional` default TRUE: expectation and variance are calculated using the conditional randomization null (Sokal 1998 Eqs. A7 & A8). Elaboration of these changes available in Sauer et al. (2021). If FALSE: expectation and variance are calculated using the total randomization null (Sokal 1998 Eqs. A3 & A4).

```
localmoran(Density, TWN_nb_w, zero.policy=T, conditional = FALSE)
```

```
             Ii   E.Ii     Var.Ii        Z.Ii Pr(z != E(Ii))
221 0.668654386 -0.025 0.17429750  1.66148935   9.661521e-02
222 0.552267439 -0.025 0.22386865  1.22005786   2.224429e-01
223 1.122969659 -0.025 0.17429750  2.74969696   5.965040e-03
224 0.520540764 -0.025 0.14125006  1.45155294   1.466260e-01
225 1.283663684 -0.025 0.14125006  3.48203973   4.976098e-04
```

-1/(n-1)

Total randomization null

```
localmoran(Density, TWN_nb_w, zero.policy=T, conditional = FALSE)

             Ii  E.Ii     Var.Ii        Z.Ii Pr(z != E(Ii))
221 0.668654386 -0.025 0.17429750 1.66148935   9.661521e-02
222 0.552267439 -0.025 0.22386865 1.22005786   2.224429e-01
223 1.122969659 -0.025 0.17429750 2.74969696   5.965040e-03
224 0.520540764 -0.025 0.14125006 1.45155294   1.466260e-01
225 1.283663684 -0.025 0.14125006 3.48203973   4.976098e-04
```

Let the "total randomization hypothesis" be one under which all permutations of the observed data values on the locations are equally likely. For the total randomization hypothesis, the moments of I_i are derived in the Mathematical Appendix. The expected value of I_i is

$$E(I_i) = -w_i/(n-1) \tag{4}$$

where w_i is the sum $\Sigma_j w_{ij}$ of all weights connected to (leaving) location i. A value of I_i above its expectation indicates a cluster of variates around locality i similar to each other and to the variate z_i. A value of I_i below its expectation signifies connected variates dissimilar to that at i. The formula for the variance $V(I_i)$ was first derived by Anselin (1995), whose formula, slightly modified, is

$$V(I_i) = w_{i(2)}(n-b_2)/(n-1) + (w_i^2 - w_{i(2)})(2b_2 - n)/[(n-1)(n-2)] \\ - [-w_i/(n-1)]^2. \tag{5}$$

https://onlinelibrary.wiley.com/doi/epdf/10.1111/j.1538-4632.1998.tb00406.x

Conditional randomization null

```
> LISA.Popn <- localmoran(Density, TWN_nb_w, zero.policy=T, conditional = TRUE)
> LISA.Popn
             Ii          E.Ii       Var.Ii        Z.Ii Pr(z != E(Ii))
221 0.668654386 -3.779349e-02 2.676102e-01  1.36561673   1.720593e-01
222 0.552267439 -2.314840e-02 2.139495e-01  1.24401656   2.134935e-01
223 1.122969659 -5.543082e-02 3.853030e-01  1.89841748   5.764111e-02
224 0.520540764 -1.030478e-02 6.075569e-02  2.15364779   3.126780e-02
225 1.283663684 -2.499016e-02 1.451527e-01  3.43488554   5.928042e-04
226 1.275561091 -6.572105e-02 4.518543e-01  1.99535859   4.600378e-02
227 2.310784759 -5.639251e-02 3.170003e-01  4.20437007   2.618103e-05
```

The local Geary coefficient c_i is defined as

$$c_i = (1/m_2)\Sigma_j w_{ij}(z_i - z_j)^2,$$

$$E(c_i) = 2nw_i/(n-1). \tag{7}$$

All terms in this expression have already been defined. The variance $V(c_i)$ is

$$V(c_i) = [n/(n-1)](w_i^2 + w_{i(2)})(3 + b_2) - [2nw_i/(n-1)]^2. \tag{8}$$

https://onlinelibrary.wiley.com/doi/epdf/10.1111/j.1538-4632.1998.tb00406.x

Local Moran's I in R: permutation test

using **localmoran_perm()** function

```
localmoran_perm(x, listw, nsim=499, zero.policy=NULL, na.action=na.fail,
    alternative = "two.sided", mlvar=TRUE,
        spChk=NULL, adjust.x=FALSE, sample_Ei=TRUE, iseed=NULL,
        no_repeat_in_row=FALSE)
```

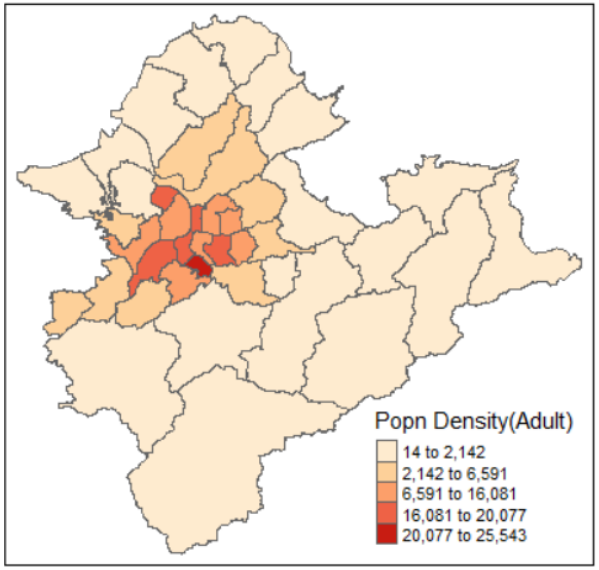
```
> LISA.Popn2 <- localmoran_perm(Density, TWN_nb_w, zero.policy=T)
> LISA.Popn2
              Ii          E.Ii       Var.Ii        Z.Ii Pr(z != E(Ii)) Pr(z != E(Ii)) Sim
221  0.668654386 -0.0669090328 2.856191e-01  1.37634251   1.687156e-01              0.160
222  0.552267439 -0.0174631722 2.542451e-01  1.12990851   2.585148e-01              0.272
223  1.122969659 -0.0417704487 4.427884e-01  1.75037432   8.005374e-02              0.120
224  0.520540764 -0.0071149112 6.877795e-02  2.01199090   4.422090e-02              0.080
225  1.283663684 -0.0512973853 1.656873e-01  3.27962397   1.039455e-03              0.008
226  1.275561091 -0.0587890733 5.539305e-01  1.79284266   7.299809e-02              0.096
227  2.310784759 -0.0661175265 3.418687e-01  4.06519747   4.799181e-05              0.004
228 -0.031403199 -0.0001949660 4.509088e-04 -1.46968696   1.416466e-01              0.172
229  0.008059578 -0.0097234513 1.902973e-02  0.12891084   8.974282e-01              0.960
230 -0.001253596  0.0002142556 4.068078e-05 -0.23013756   8.179849e-01              0.744
231 -0.044515937 -0.0069749599 1.977674e-02 -0.26694894   7.895085e-01              0.752
```

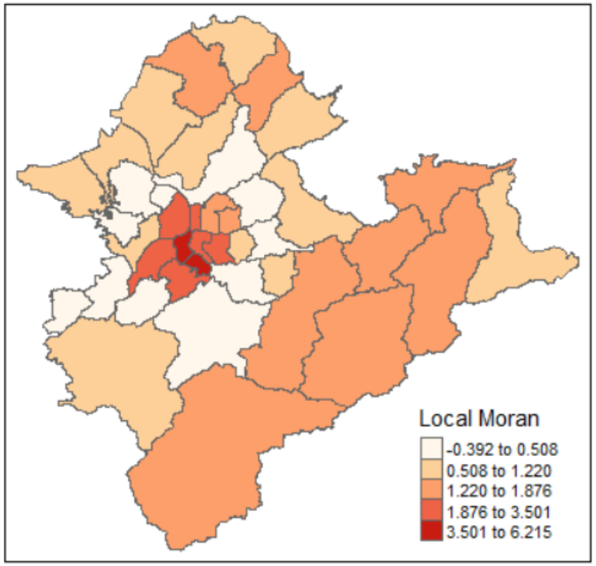
R Lab: Local Moran's I in R

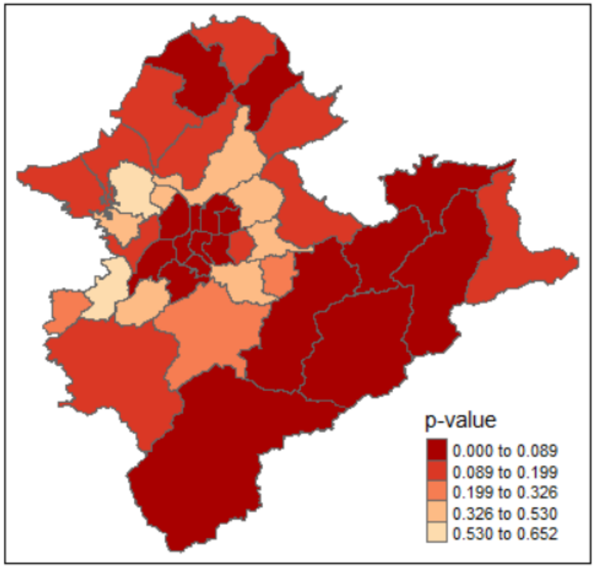
Population Density

Local Moran (z-score for LISA)

p-value







attr: Object Attributes

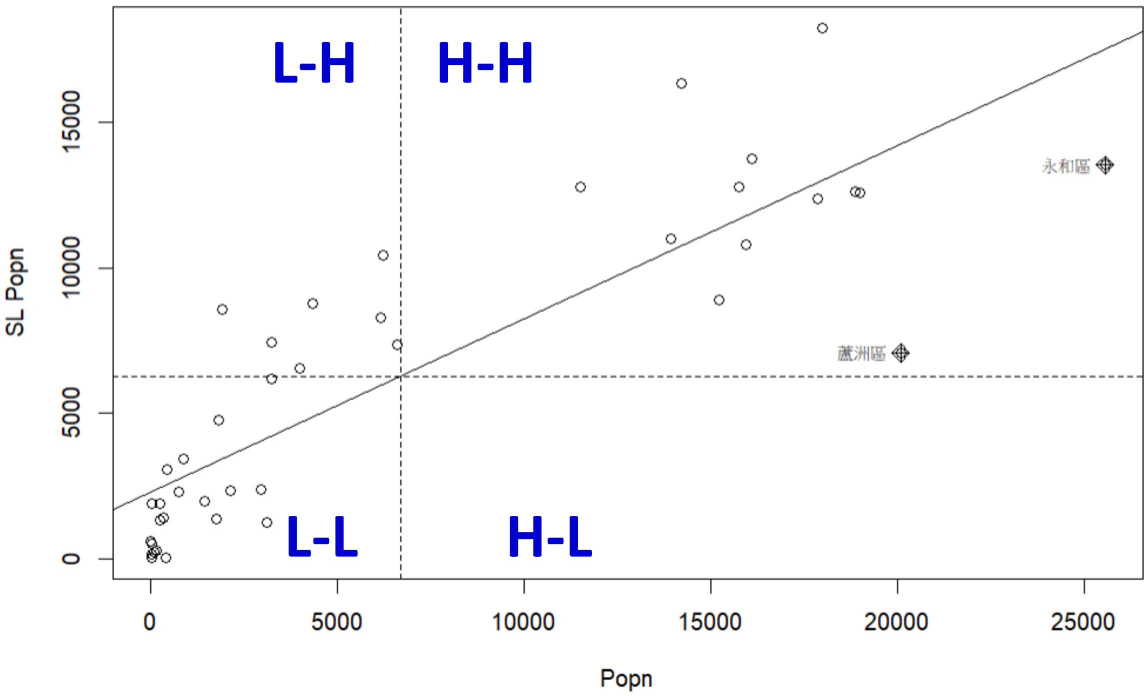
Description

Get or set specific attributes of an object.

```
attr(,"quadr")
         mean    median     pysal
1   High-High High-High High-High
2   High-High High-High High-High
3   High-High High-High High-High
4   High-High High-High High-High
5   High-High High-High High-High
6   High-High High-High High-High
7   High-High High-High High-High
8    Low-High High-High  Low-High
9    Low-High High-High   Low-Low
```

Moran Scatter plot and Local Moran

moran.plot (Density, TWN_nb_w, labels=IDs , xlab="Popn", ylab="SL Popn")




```
> quadr <- attr(LISA.Popn, "quadr")$mean
> quadr
 [1] High-High High-High High-High High-High High-High High-High High-High Low-High  Low-High
[10] Low-High  Low-High  Low-Low   High-High High-High High-High High-High High-High Low-Low
[19] Low-High  Low-Low   Low-Low   Low-Low   Low-Low   Low-Low   Low-High  High-High Low-High
[28] Low-Low   Low-Low   Low-Low   Low-Low   Low-Low   Low-Low   Low-Low   Low-Low   Low-Low
[37] Low-Low   Low-Low   Low-Low   Low-Low   Low-Low
Levels: Low-Low High-Low Low-High High-High
```

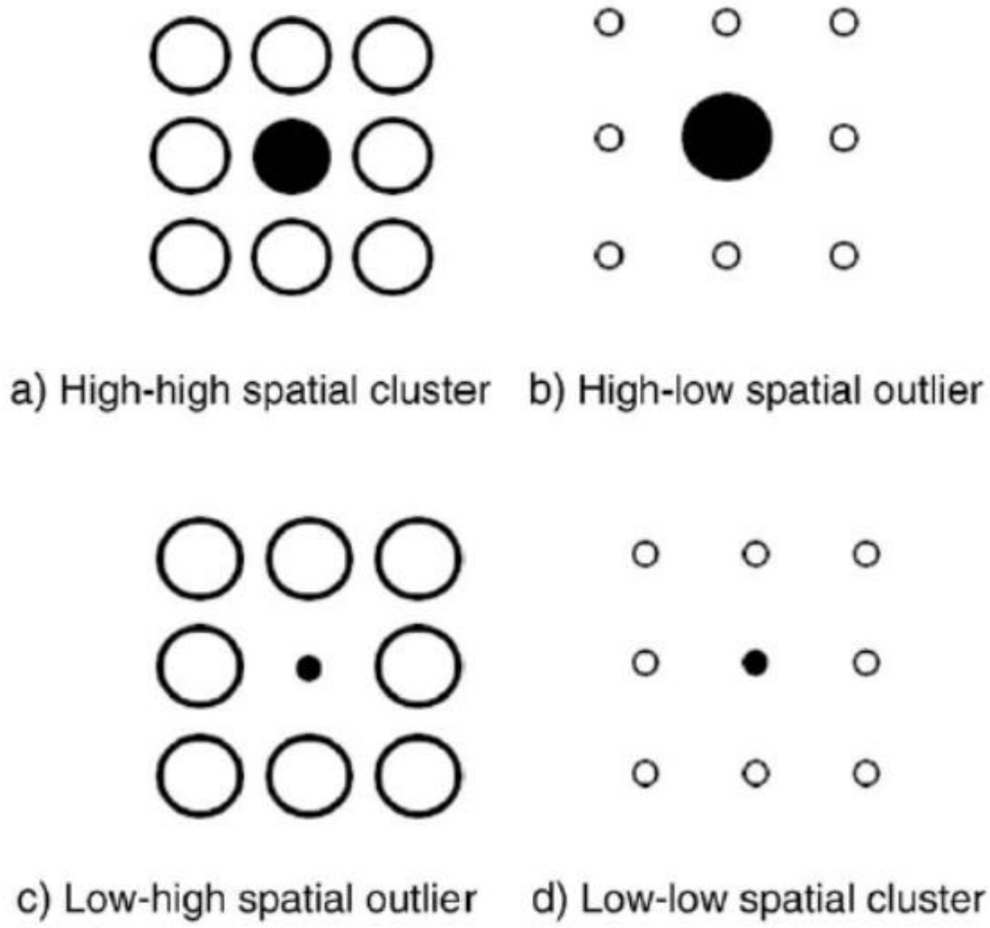
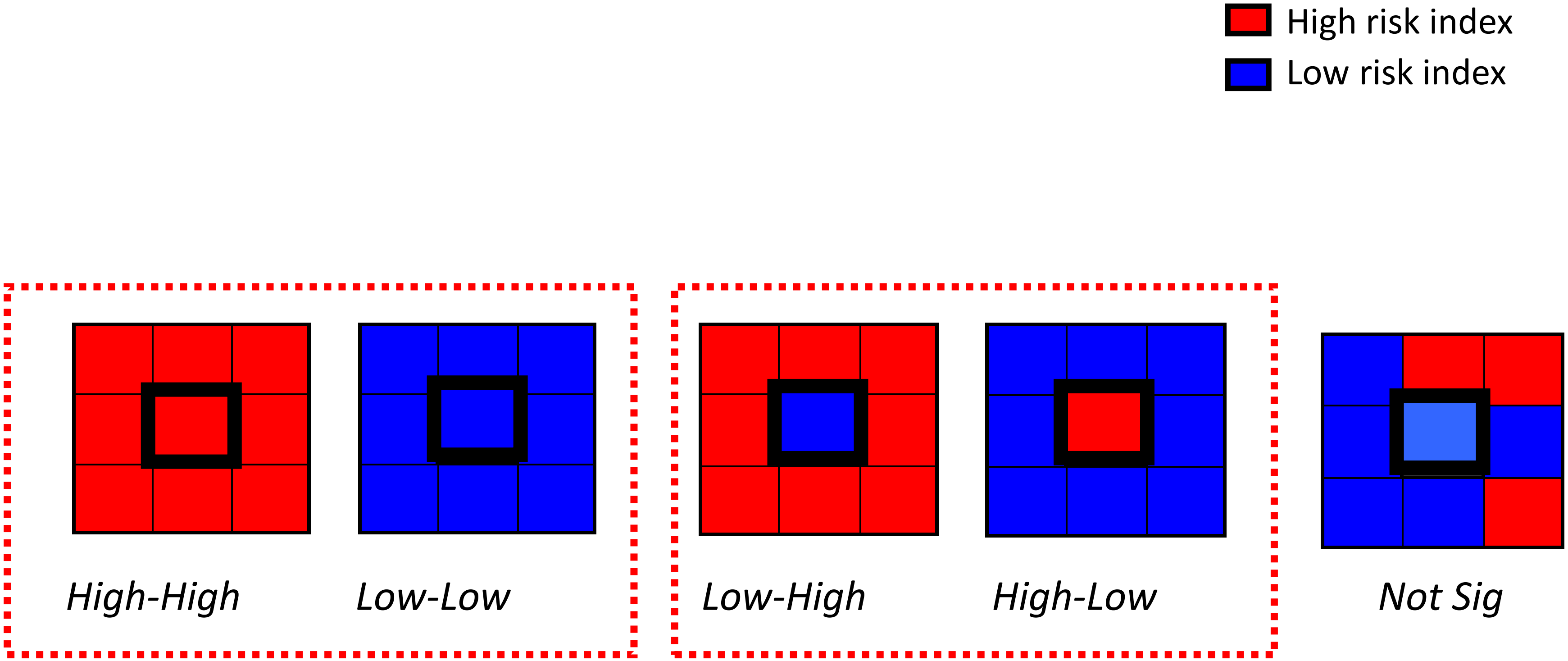


Fig. 1 – Sketch figure showing the relationship of a location and its neighbourhood: a) and d) spatial cluster; b) and c) spatial outlier; a) and b) hot spots; c) and d) cool spots.

Zhang et al., 2008

Local Moran's I (LISA)

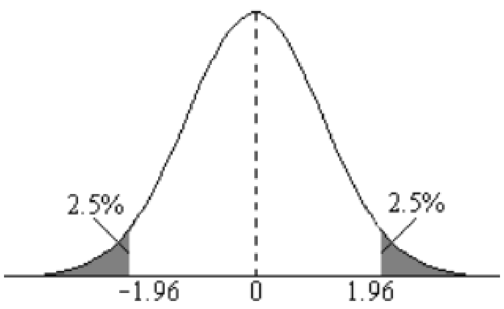


High LISA value:
Cluster of similar values

Low LISA value:
Cluster of dissimilar values

R code (new)

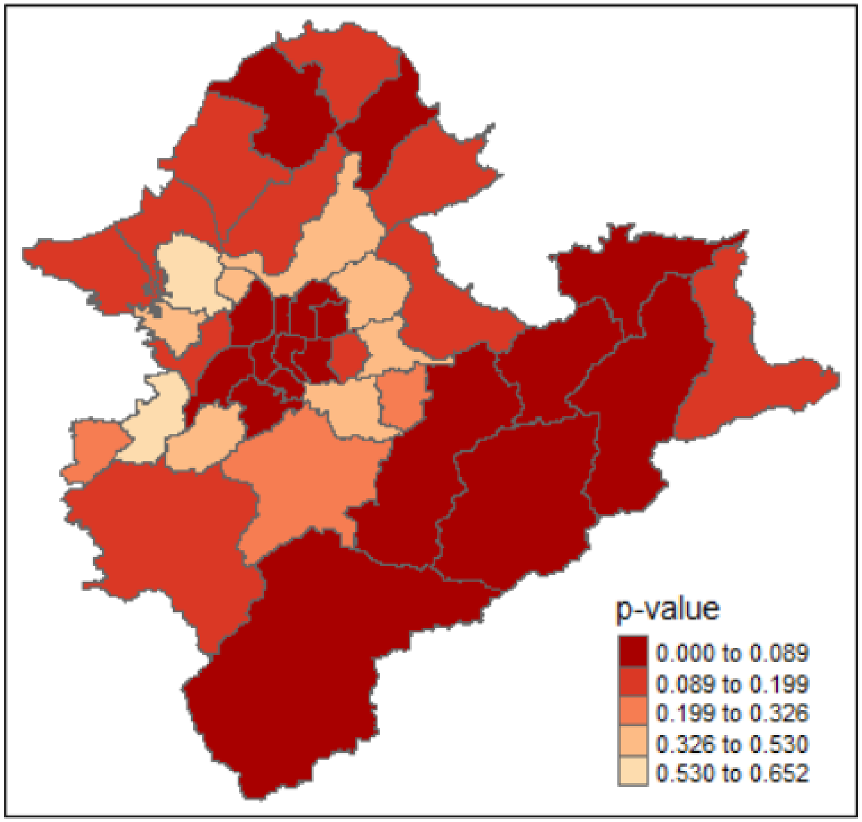
```
quadr <- attr(LISA.Popn, "quadr")$mean
quadr <- factor(quadr, levels = c(levels(quadr), "NoSig"))
NorthTW_sf$Type <- quadr
head(NorthTW_sf)
table(NorthTW_sf$Type)

signif <- 0.05
quadr[LISA.Popn[, 5]> signif] <- "NoSig"
NorthTW_sf$Type <- quadr
```



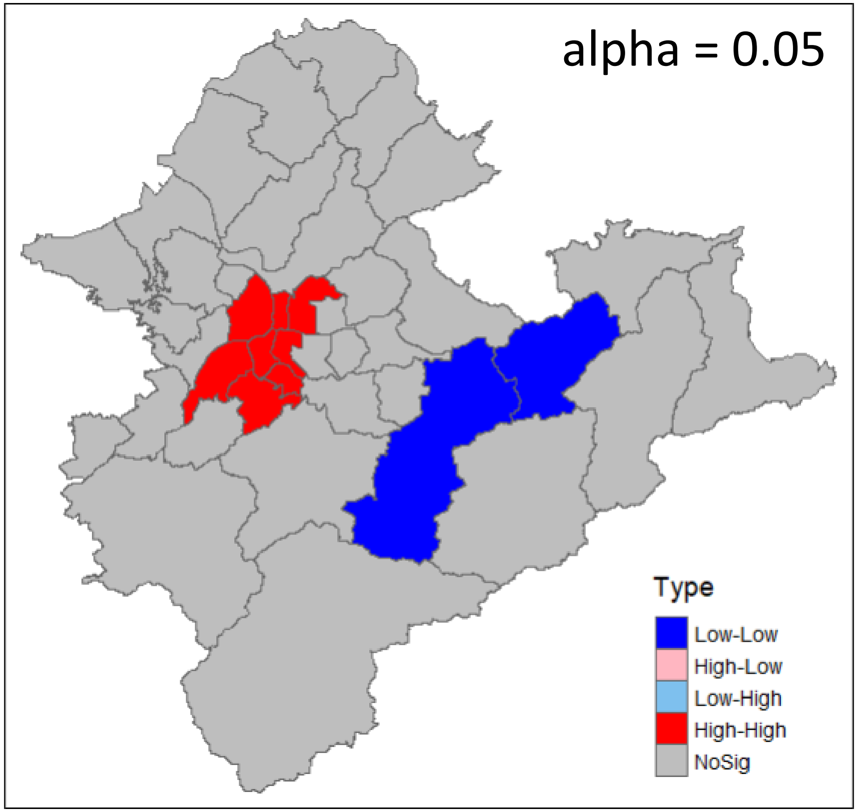

```
colors<- c( 'High-High' ='red', 'Low-Low'='blue',
            'High-Low'='lightpink', 'Low-High'='skyblue2', 'NoSig'='grey')

tm_shape(NorthTW_sf) + tm_polygons("Type", palette = colors)
```

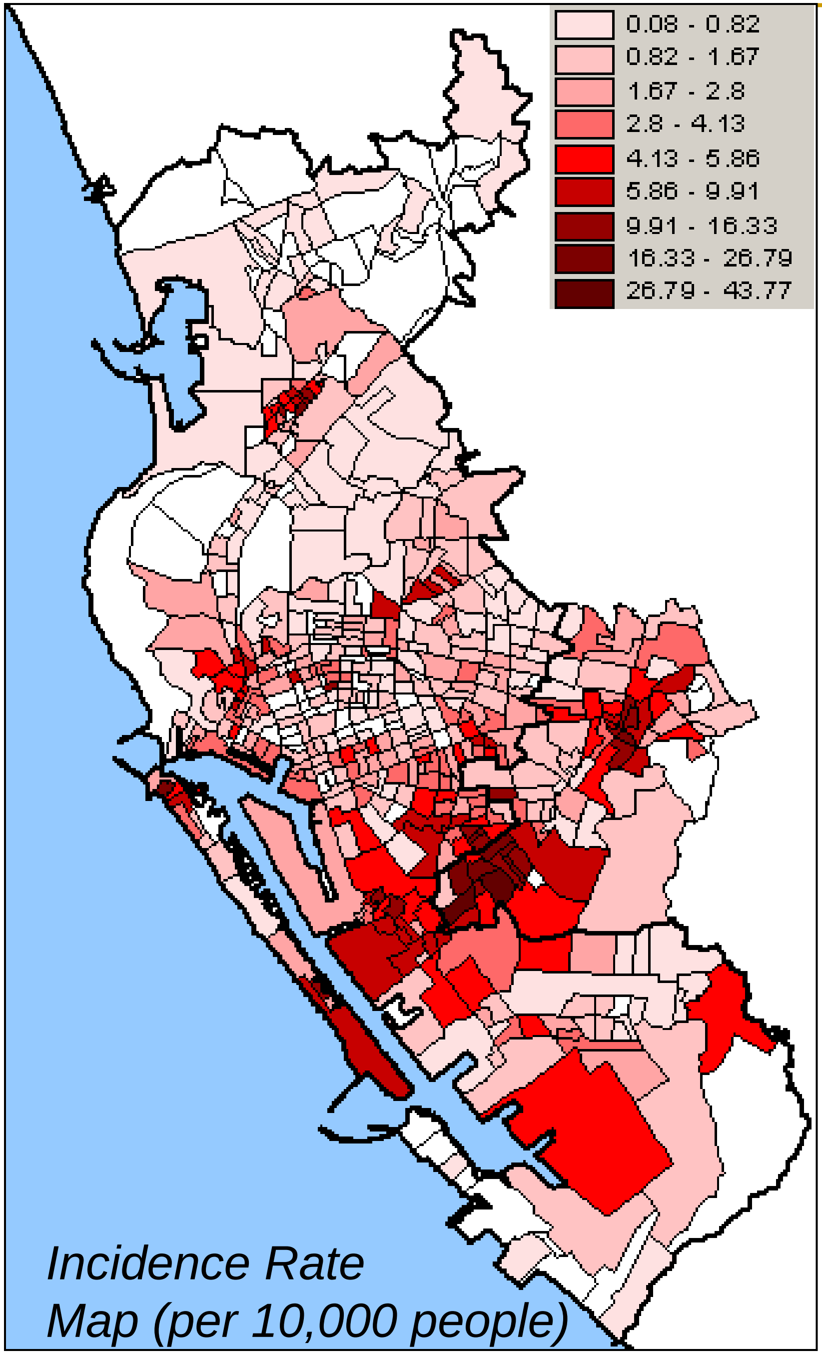
R Lab: Local Moran's I

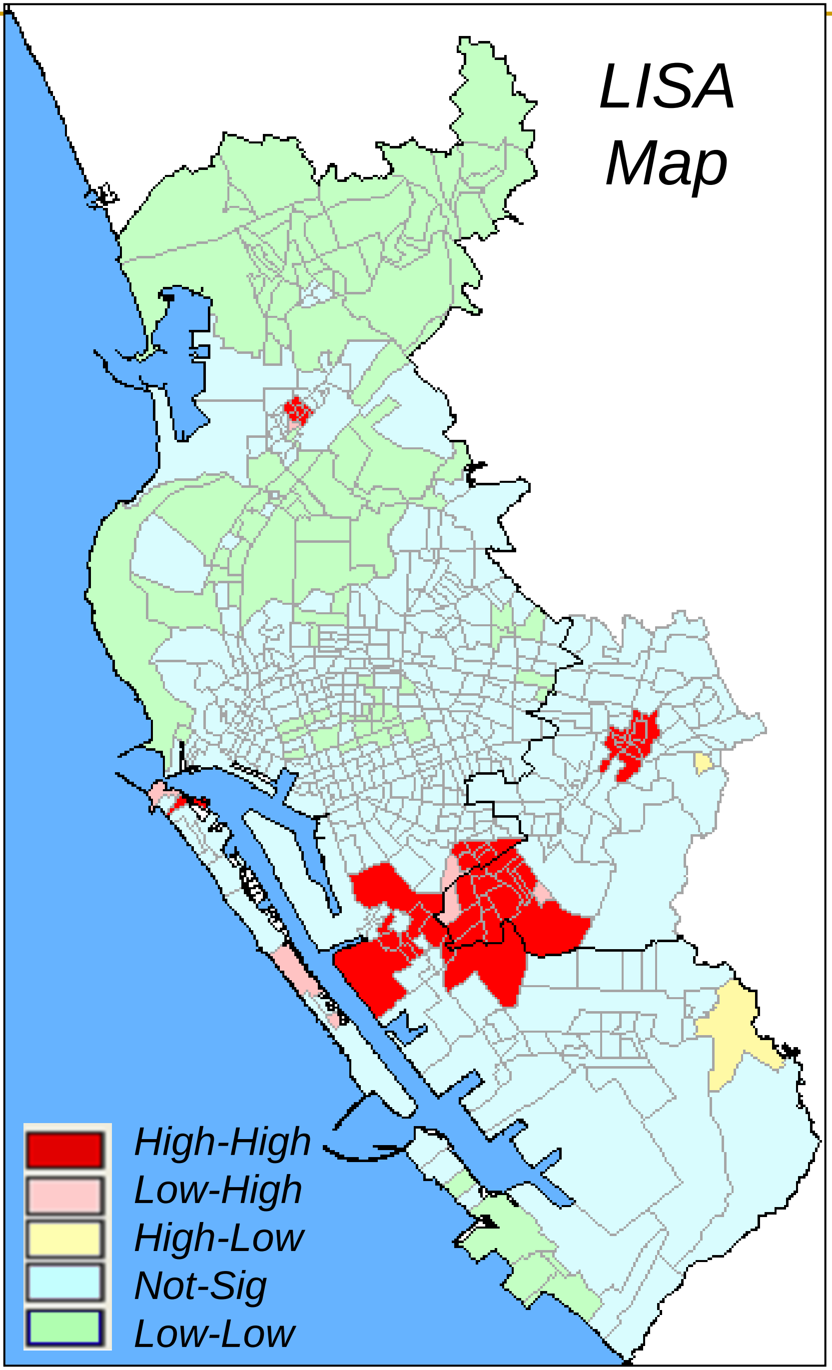
p-value of Local Moran



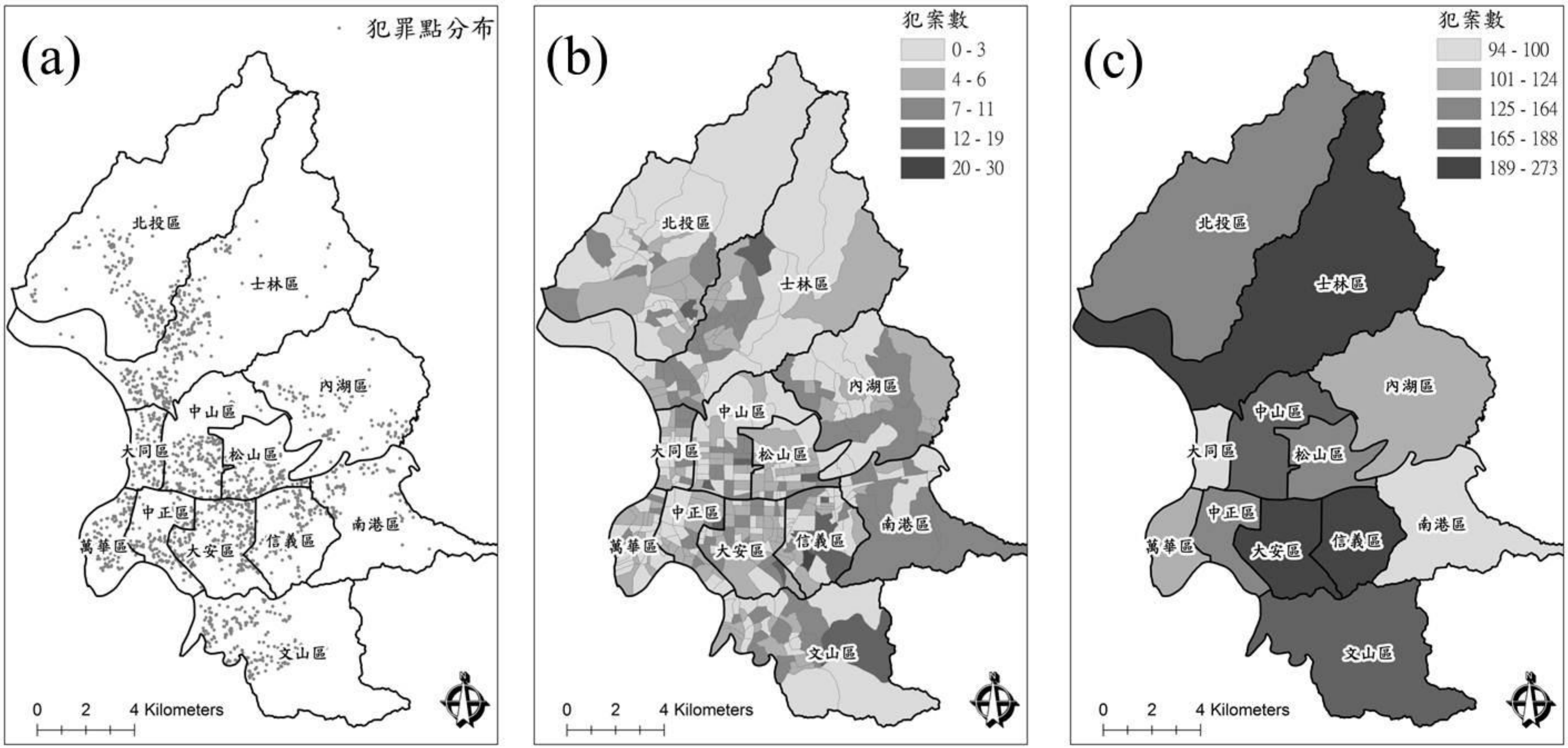
LISA Cluster Map




0.08 - 0.82
0.82 - 1.67
1.67 - 2.8
2.8 - 4.13
4.13 - 5.86
5.86 - 9.91
9.91 - 16.33
16.33 - 26.79
26.79 - 43.77
Incidence Rate
Map (per 10,000 people)

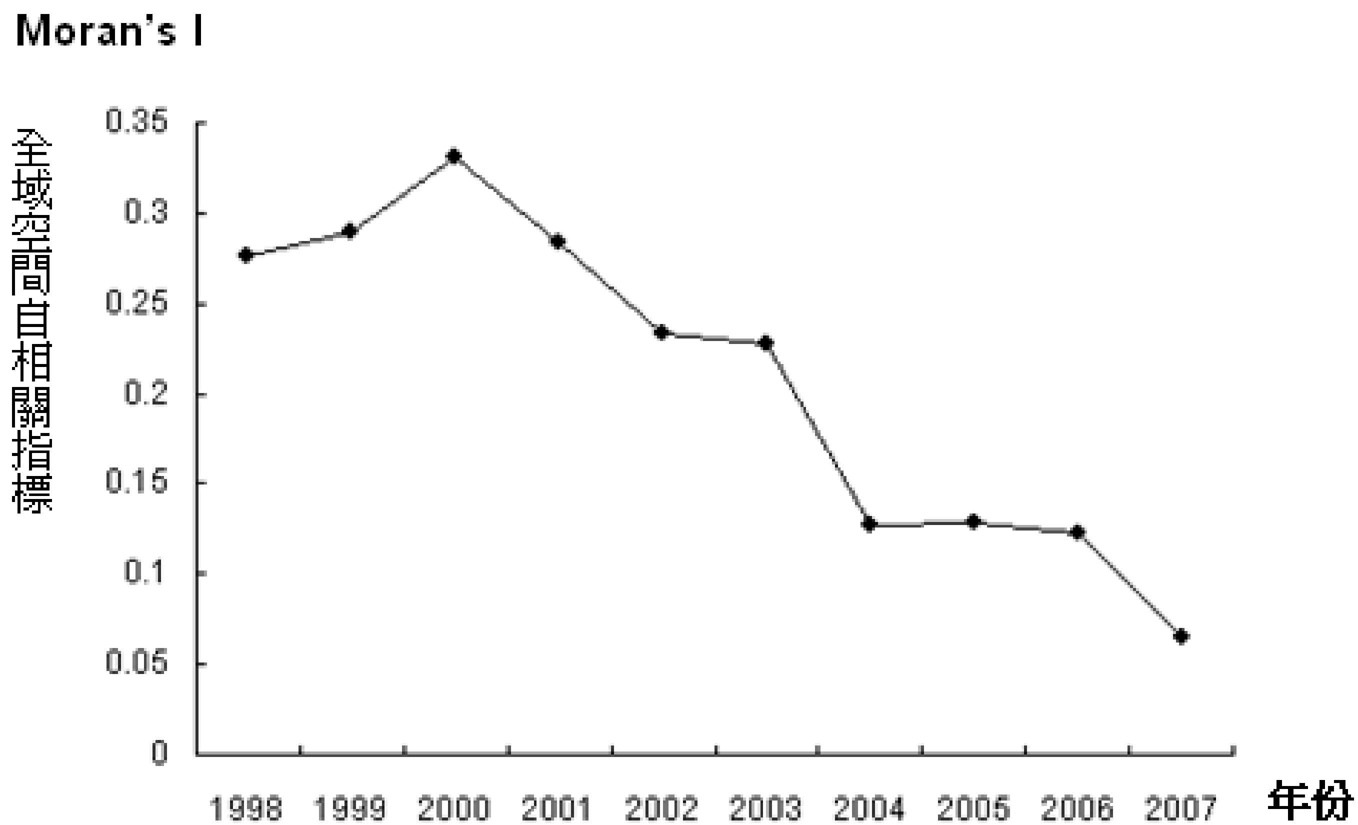

LISA
Map
High-High
Low-High
High-Low
Not-Sig
Low-Low

應用範例：
1998-2007年台北市住宅竊盜犯罪趨勢分析

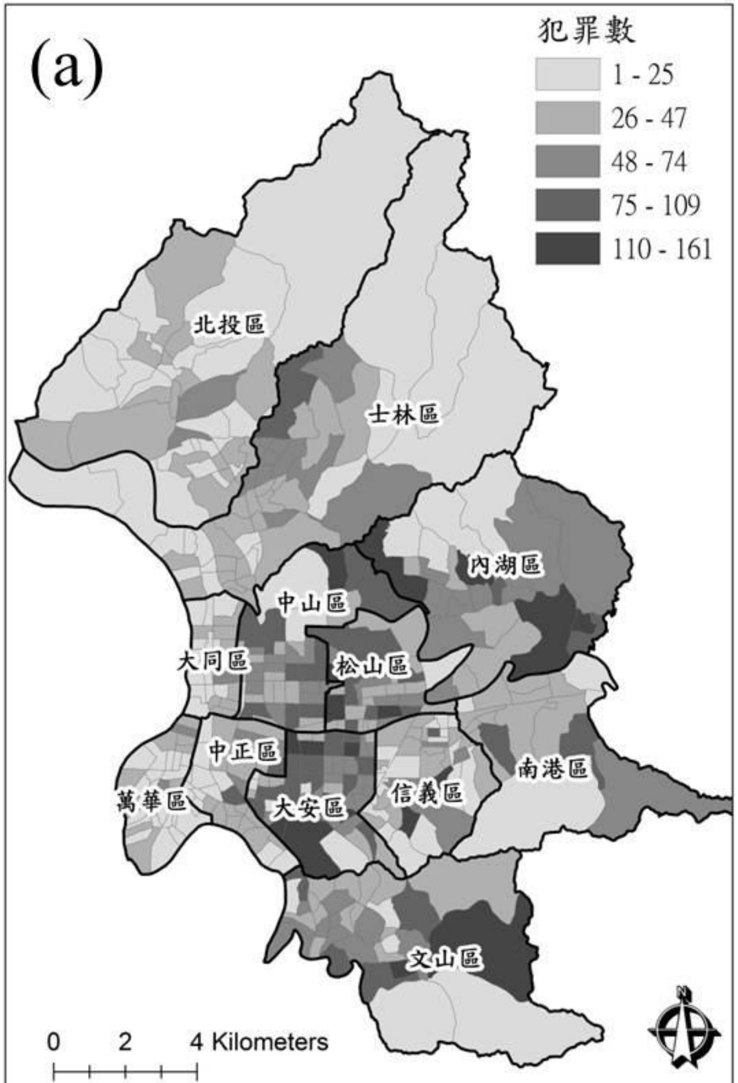


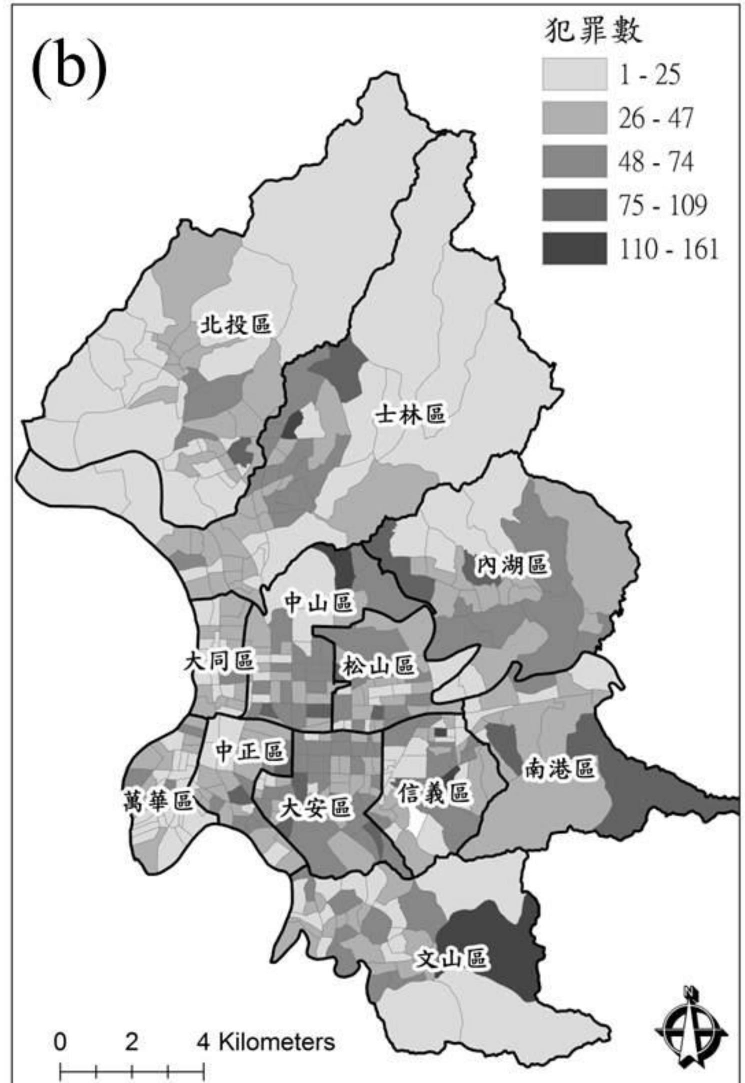
溫在弘等 (2010)，犯罪地圖繪製與熱區分析方法及其應用：以1998~2007年臺北市住宅竊盜犯罪為例，地理研究 52:43-63

Temporal Trend of Global Moran's I

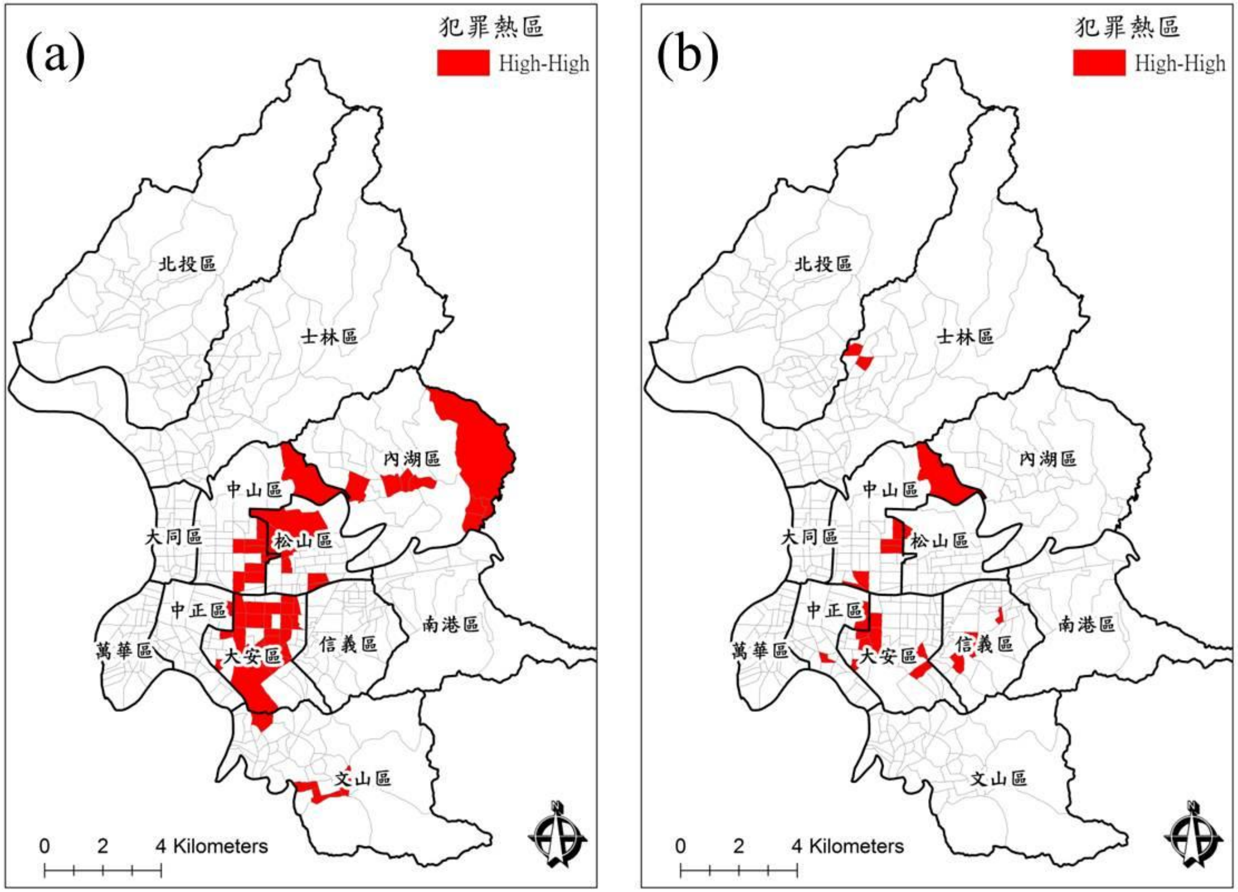


(a). 1998-2002 vs. (b). 2003-2007





Local Moran's I (LISA): H-H hot-spots
(a). 1998-2002 vs. (b). 2003-2007



參數設定：以正方格四交點相鄰的Queen型態為相鄰定義，紅色的地區亦即表示在0.05的統計顯著水準下，犯罪趨勢顯著呈現地理群聚的區域

Recap: General G-statistic

- Moran's I & Geary's C Ratio 無法區別

  " hot spots" or "cold spots"

- *Spatial Concentration* method
- Definition

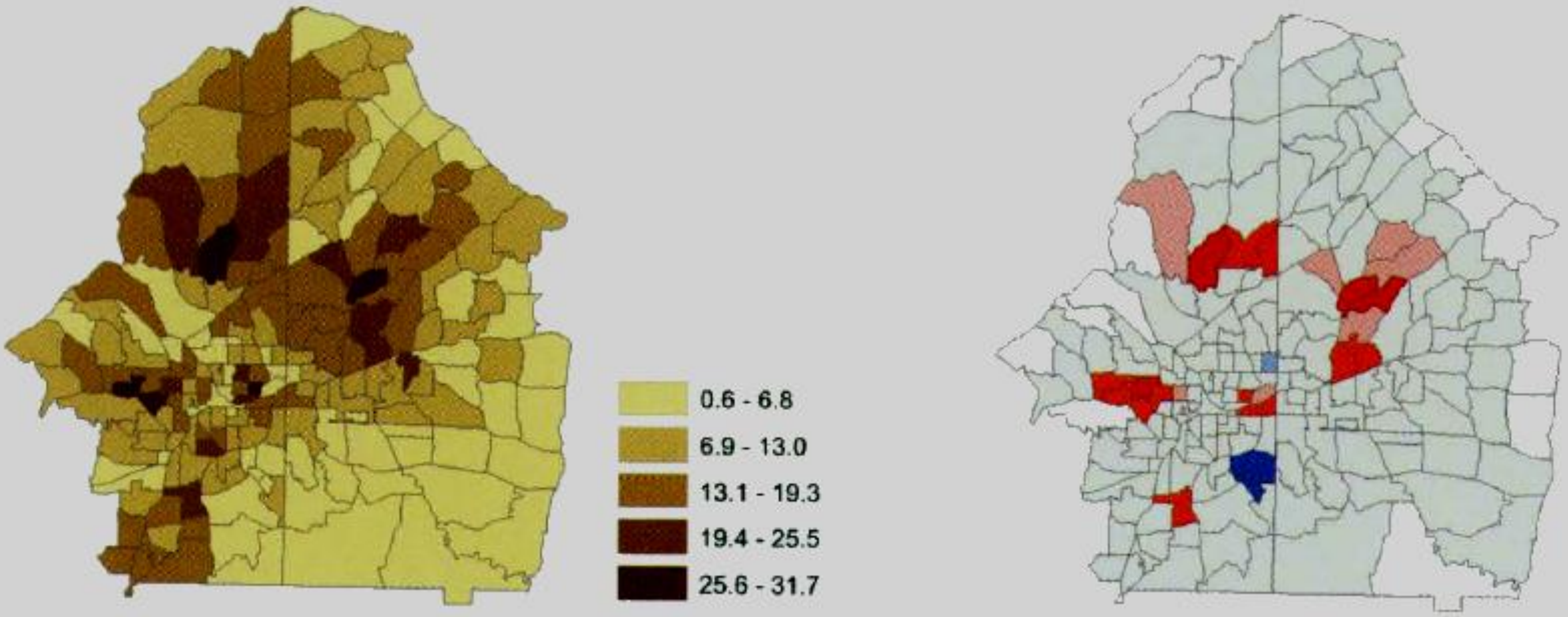
$$G(d) = \frac{\sum \sum w_{ij}(d) x_i x_j}{\sum \sum x_i x_j}$$

d : neighborhood distance

W_{ij} : 1 if it is within d, 0 otherwise

- Calculation of G must begin by identifying a neighborhood distance within which cluster is expected to occur

2. Local Analysis of G-statistic: Identifying spatial concentration with low and high values



Percent age 65 and over, by census tract (left). The map on the right shows clusters of tracts having a high percentage of seniors (orange) and tracts having a significantly lower percentage than their neighbors (blue).

Source: Chapter 4 Identifying clusters
The ESRI Guide to GIS Analysis, Volume 2

Local G-statistic

Two versions of the local G-statistic

There are two versions of this statistic, both developed by Art Getis and Keith Ord. In one version, the value of the target feature itself is not included in the equation. This is the Gi statistic. You'd use the Gi statistic if you're interested in the effect of the target feature on what's going on around it. This would be the case if you're interested in the dispersion of a particular phenomenon from the target feature to the surrounding area over time. Getis and Ord, for example, used Gi to track the dispersion of AIDS to counties surrounding San Francisco County over the course of several years. They wanted to see if the intensity of clustering of AIDS cases in counties surrounding San Francisco increased over time and the distance at which the clustering peaked. See the references at the end of this chapter for more on the Gi statistic.

In the other version, called Gi* (pronounced G-i-star), the value of the target feature is included. If you're interested in finding hot spots or cold spots, you'd use Gi*—you'll want to include the value of the target feature since its value contributes to the occurrence of the cluster.

Source: Chapter 4 Identifying clusters
The ESRI Guide to GIS Analysis, Volume 2

Getis-Ord Local G Statistic

Neighborhood Definition

$$G_i(d) = \frac{\sum_j w_{ij}(d) x_j}{\sum_j x_j};\ j \neq i$$

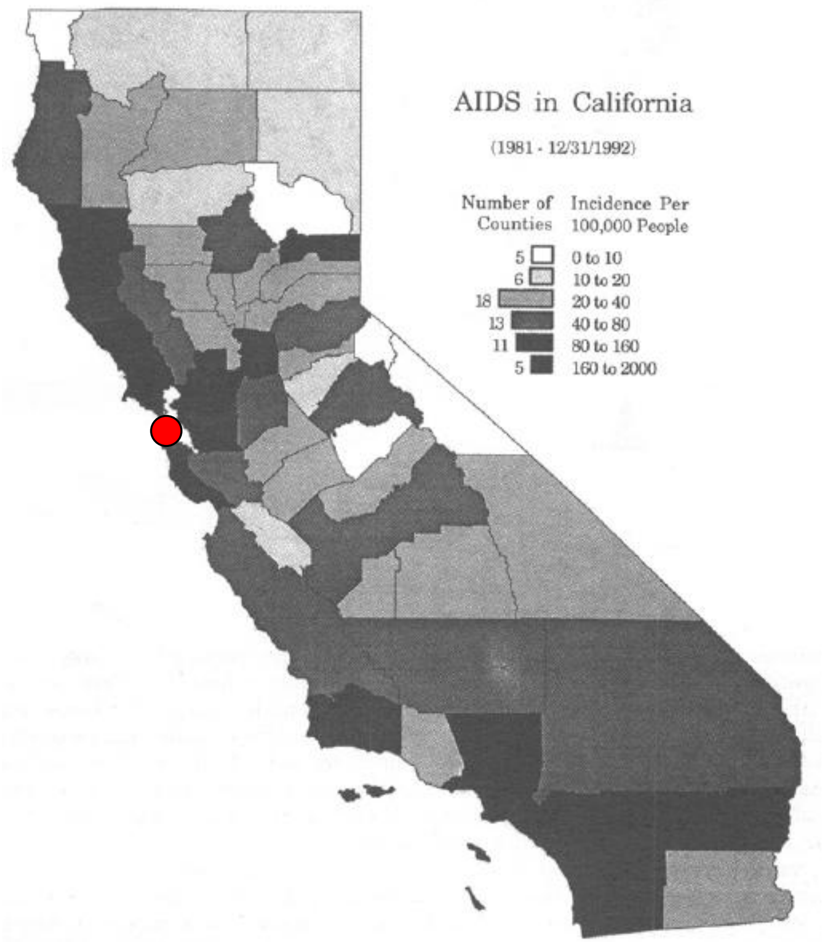
- The Gi statistic **excludes** the value at i from the summation and is used for spread or diffusion studies
- the Gi* **includes** the value at i in the summation (for all j) and is most often used for studies of clustering

Smaller ← Gi(d) → *Larger*

Cluster of low values Mean *Cluster of high values*

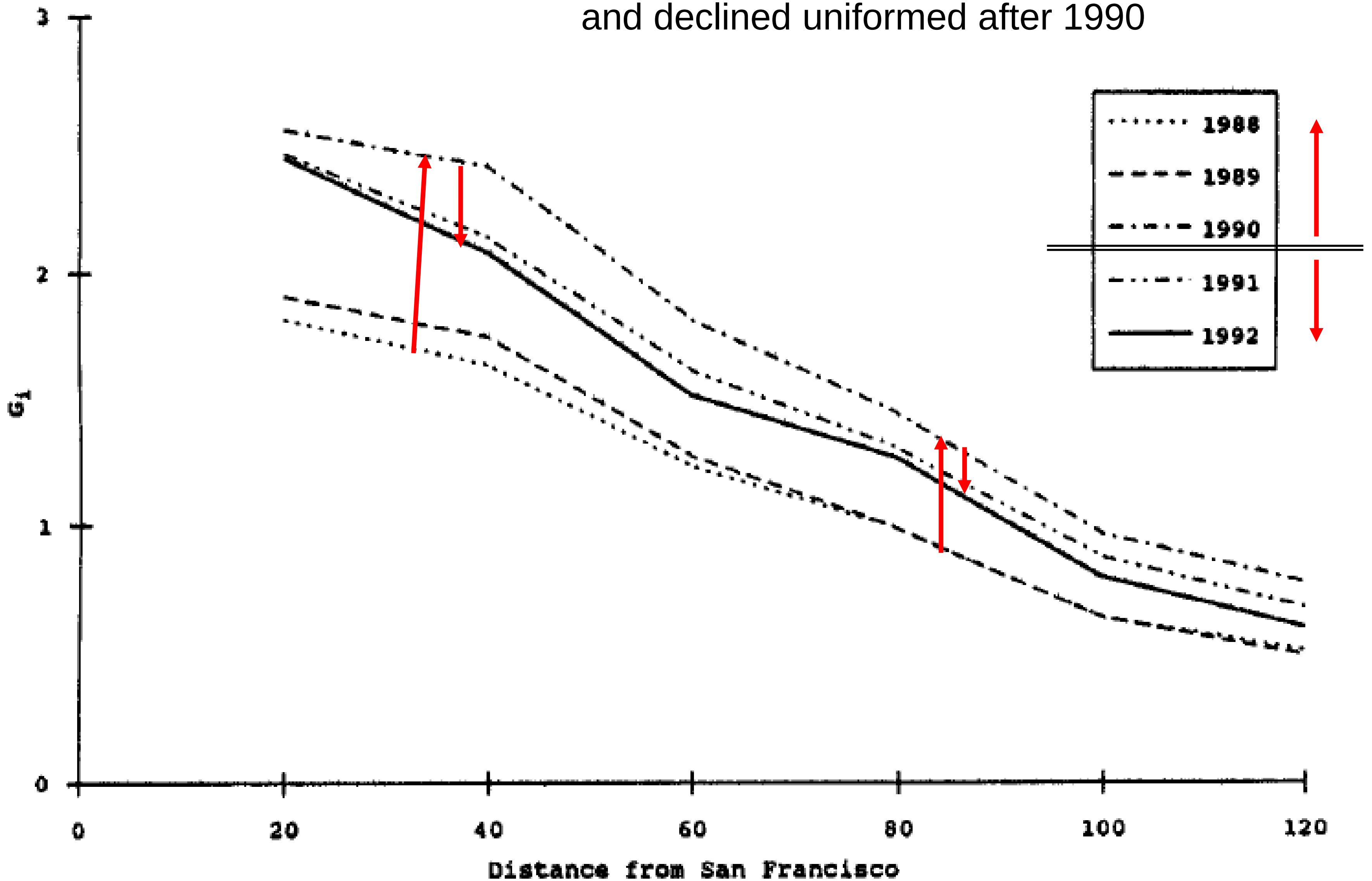
Source: Chapter 4 Identifying clusters
The ESRI Guide to GIS Analysis, Volume 2

Example: Diffusion of AIDS in California



Source: Ord and Getis,1995

The rate of AIDS cases increased uniformly over the area of clustering from 1988-1990 and declined uniformed after 1990



Source: Ord and Getis,1995

Testing the statistical significance of Gi*

The Z-score for Gi*

The expected Gi* value is subtracted from the observed Gi*....

$$Z(G_i^*) = \frac{G_i^* - E(G_i^*)}{\sqrt{Var(G_i^*)}}$$

....and the difference divided by the square root of the variance

The expected Gi* value

The weights (w_{ij}) at a distance (d) are summed....

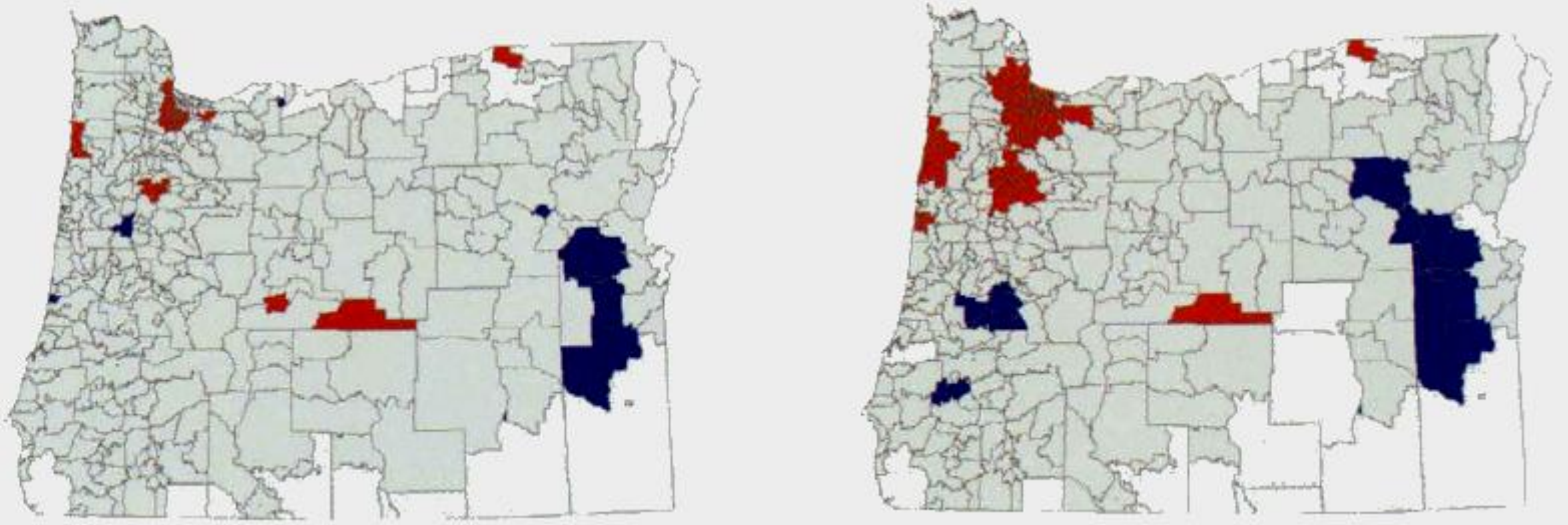
$$E(G_i^*) = \frac{\sum_{i} w_{ij}(d)}{n-1}$$

....and divided by the number of features (n), minus one

Source: Chapter 4 Identifying clusters
The ESRI Guide to GIS Analysis, Volume 2

Neighborhood Definition

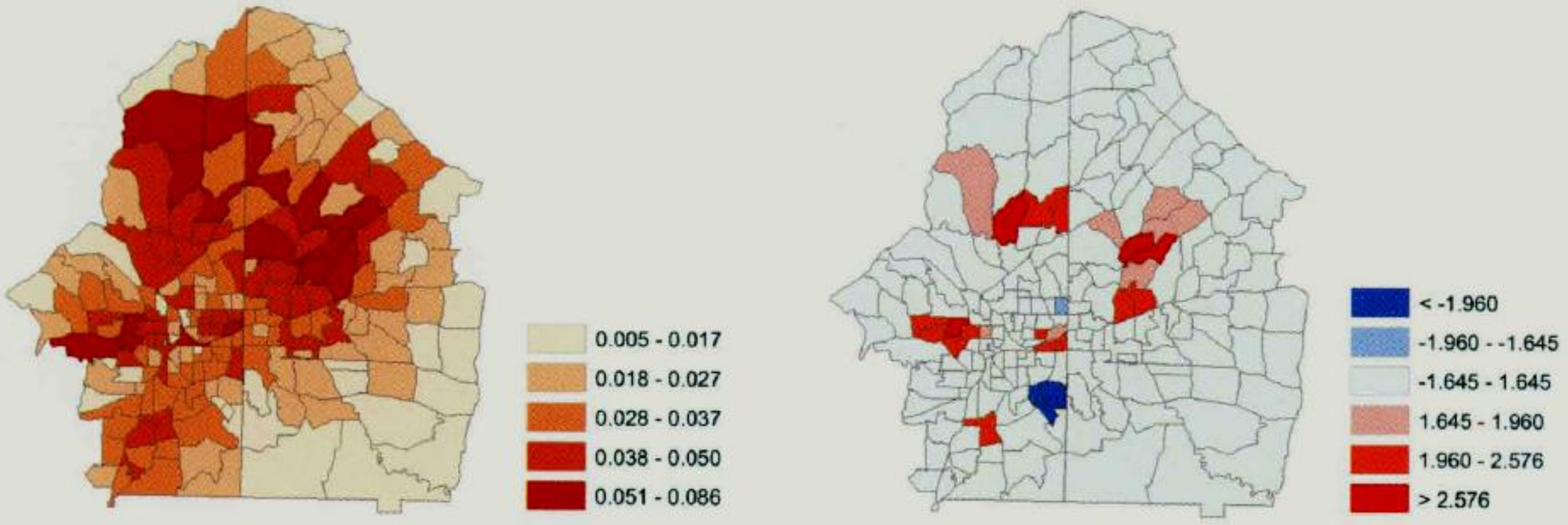
- Distance-based neighborhood.



Clusters of ZIP Codes having high numbers of people more likely (orange) or less likely (blue) to buy pet supplies. Using a distance of five miles (left map), the clusters are smaller and more localized. Using a distance of 20 miles (right) creates larger, regional clusters.

Source: Chapter 4 Identifying clusters
The ESRI Guide to GIS Analysis, Volume 2

Mapping the result of Gi* values



Census tracts color coded by Gi values (left) and Z-scores, calculated from percent age 65 and over*

Source: Chapter 4 Identifying clusters
The ESRI Guide to GIS Analysis, Volume 2

R Lab: Using R packages to calculate Gi*

using **localG()** function

G and Gstar local spatial statistics

Description

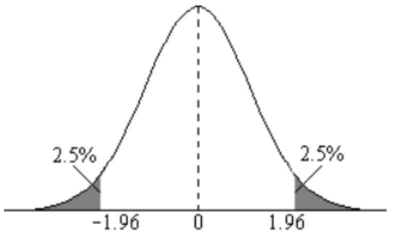
The local spatial statistic G is calculated for each zone based on the spatial weights object used. The value returned is a Z-value, and may be used as a diagnostic tool. High positive values indicate the posibility of a local cluster of high values of the variable being analysed, very low relative values a similar cluster of low values. For inference, a Bonferroni-type test is suggested in the references, where tables of critical values may be found (see also details below).

Usage

```
localG(x, listw, zero.policy=NULL, spChk=NULL, GeoDa=FALSE, alternative = "two.sided",
 return_internals=TRUE)
```

R Lab: Using R packages to calculate Gi*

using **localG()** function




```
localG(x, listw, zero.policy=NULL, spChk=NULL, GeoDa=FALSE, alternative = "two.sided",
 return_internals=TRUE)
```

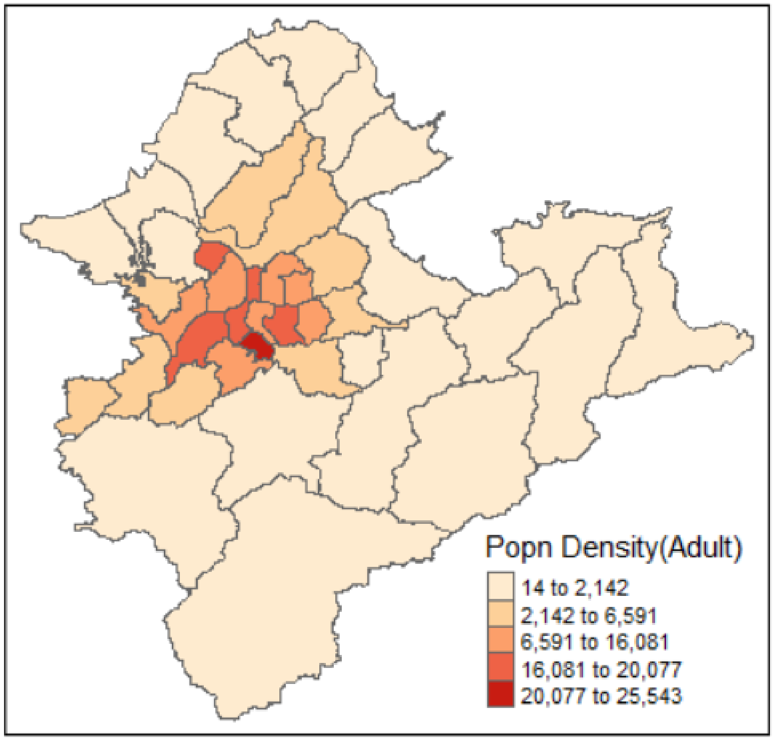
```
TWN_nb_in <- include.self(TWN_nb)

TWN_nb_in_w <- nb2listw(TWN_nb_in, zero.policy=T)

LG <- localG(Density, TWN_nb_in_w)
```

Standardized Gi* values

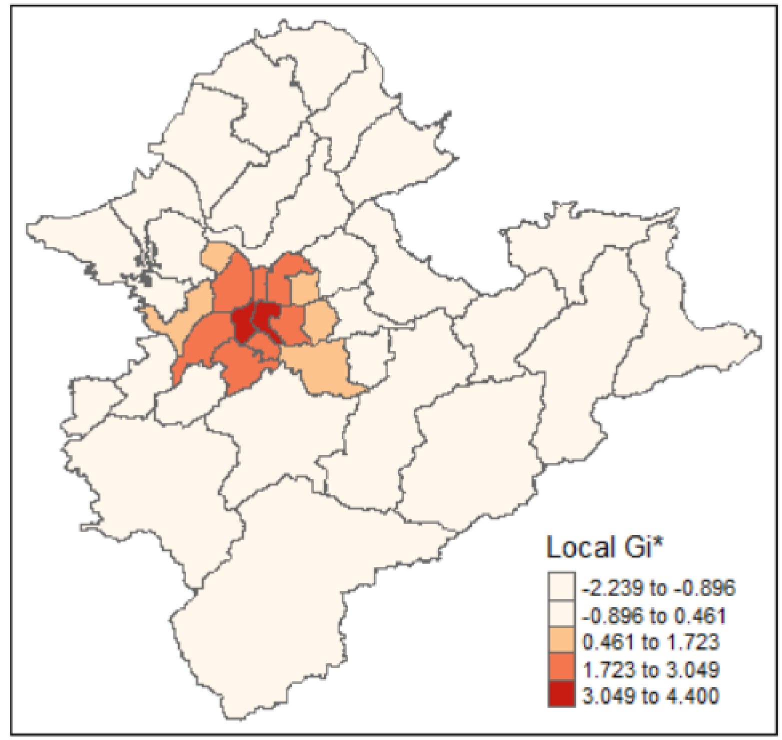
```
> LG
 [1]  1.7233446  1.5358246  2.2955770  2.2575385  3.5680055  2.4243832  4.4002452  1.5935843 -0.2011367  0.1873412
[11]  0.2039434 -1.7879950  2.6015773  3.0486377  2.4917352  2.9034051  1.1389338 -0.8962848  0.4607859 -0.9741356
[21] -1.4210311 -1.4498460 -1.6708723 -1.8162000  0.4273101  1.0063072  0.4332790 -0.3465971 -1.3485541 -1.0850525
[31] -2.2394723 -2.0938802 -1.7500732 -1.5259770 -1.6278155 -2.1806525 -2.0660263 -1.5440033 -1.9083605 -1.4421784
[41] -1.9553767
attr(,"gstari")
[1] TRUE
attr(,"call")
localG(x = Density, listw = TWN_nb_in_w)
attr(,"class")
[1] "localG"
```

R Lab: Mapping Standardized Gi* values

Population



Standardized Gi* values



Statistical Test
for Significant Hot-spots

The Z-score for Gi*

The expected Gi* value is subtracted from the observed Gi*....

$$Z(G_i^*) = \frac{G_i^* - E(G_i^*)}{\sqrt{Var(G_i^*)}}$$

....and the difference divided by the square root of the variance

The expected Gi* value

The weights (w_{ij}) at a distance (d) are summed....

$$E(G_i^*) = \frac{\sum_i w_{ij}(d)}{n-1}$$

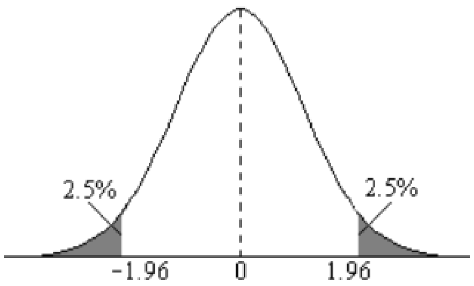
....and divided by the number of features (n), minus one

```
attr(,"internals")
               G*i    E(G*i)       V(G*i)     Z(G*i) Pr(z != E(G*i))
 [1,] 0.0423710270 0.02439024 1.088612e-04  1.7233446    8.482623e-02
 [2,] 0.0421929468 0.02439024 1.343658e-04  1.5358246    1.245814e-01
 [3,] 0.0483415035 0.02439024 1.088612e-04  2.2955770    2.170008e-02
 [4,] 0.0458835729 0.02439024 9.064358e-05  2.2575385    2.397445e-02
 [5,] 0.0583601259 0.02439024 9.064358e-05  3.5680055    3.597090e-04
 [6,] 0.0496854224 0.02439024 1.088612e-04  2.4243832    1.533442e-02
 [7,] 0.0662836240 0.02439024 9.064358e-05  4.4002452    1.081286e-05
 [8,] 0.0365111192 0.02439024 5.785193e-05  1.5935843    1.110292e-01
 [9,] 0.0226255011 0.02439024 7.698039e-05 -0.2011367    8.405917e-01
[10,] 0.0263448971 0.02439024 1.088612e-04  0.1873412    8.513931e-01
[11,] 0.0257598513 0.02439024 4.509962e-05  0.2039434    8.383977e-01
[12,] 0.0057349205 0.02439024 1.088612e-04 -1.7879950    7.377682e-02
```

R Lab: Mapping Significant Hot-spots

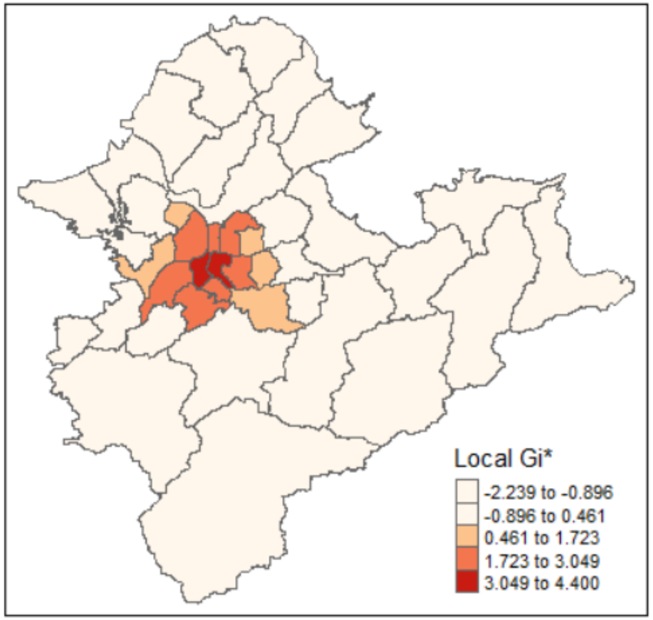
```
cluster_type <- attr(LG, "cluster")
cluster_type <- factor(cluster_type,
                       levels = c(levels(cluster_type), "NoSig"))
NorthTW_sf$CLUSTER <- cluster_type
head(NorthTW_sf)
table(NorthTW_sf$CLUSTER)

signif <- 0.05

pvalue <- attr(LG,"internals")[,5]

cluster_type[ pvalue > signif] <- "NoSig"
NorthTW_sf$CLUSTER <- cluster_type


colors <- c('High' ='red','Low' ='blue','NoSig' = 'lightgray')
tm_shape(NorthTW_sf) + tm_polygons("CLUSTER", palette = colors)
```

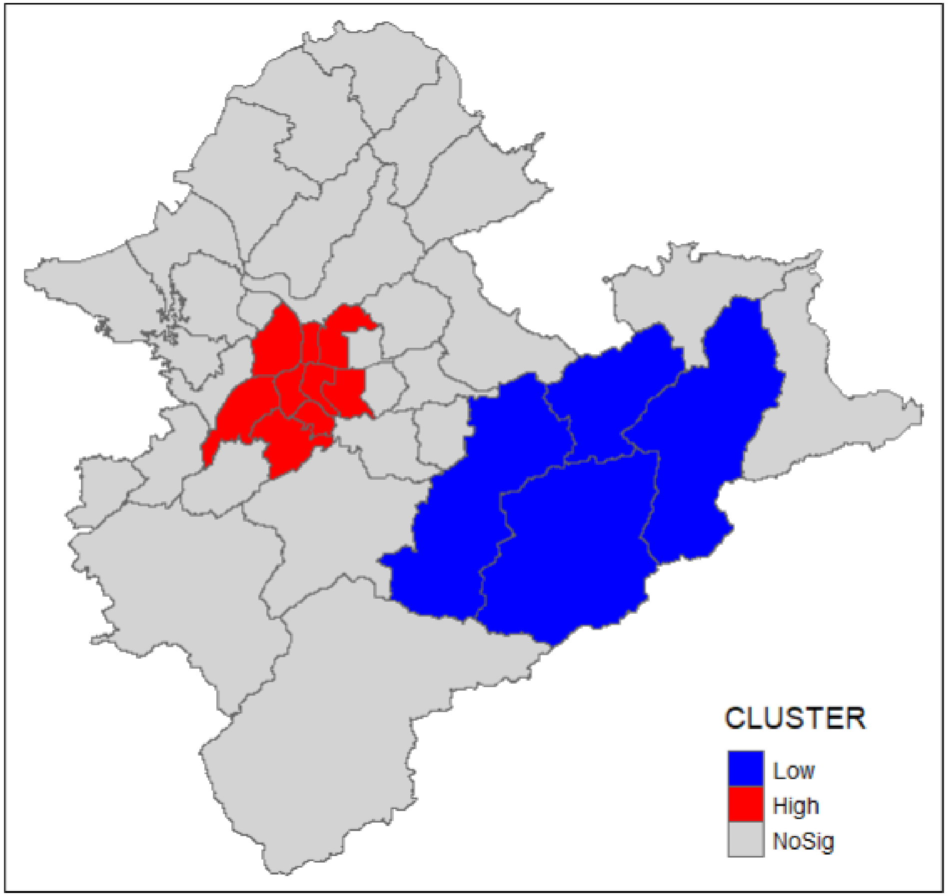
Mapping Significant Hot-spots



alpha = 0.05

Standardized Gi* values





Permutation test for Gi*:
using localg_perm() function

```
localG_perm(x, listw, nsim=499, zero.policy=NULL, spChk=NULL,
 alternative = "two.sided", iseed=NULL, fix_i_in_Gstar_permutations=TRUE,
 no_repeat_in_row=FALSE)
```

```
LG2 <- localG_perm(Density, TWN_nb_in_w, zero.policy=T)
```

```
attr(,"internals")
                Gi       E.Gi       Var.Gi   StdDev.Gi Pr(z != E(Gi)) Pr(z != E(Gi)) Sim
 [1,] 0.0423710270 0.02878989 9.317423e-05  1.40698078   0.1594330891              0.180
 [2,] 0.0421929468 0.02865076 1.242791e-04  1.21475820   0.2244583266              0.256
 [3,] 0.0483415035 0.03060241 9.986728e-05  1.77508738   0.0758834603              0.100
 [4,] 0.0458835729 0.02686970 9.527768e-05  1.94793785   0.0514224003              0.072
 [5,] 0.0583601259 0.02704150 8.461074e-05  3.40478727   0.0006621561              0.004
 [6,] 0.0496854224 0.03087008 1.095546e-04  1.79761526   0.0722379985              0.080
 [7,] 0.0662836240 0.02954181 9.664297e-05  3.73745079   0.0001858955              0.004
 [8,] 0.0365111192 0.02410787 6.523618e-05  1.53564651   0.1246251017              0.160
 [9,] 0.0226255011 0.02325691 8.668502e-05 -0.06781694   0.9459313626              0.968
[10,] 0.0263448971 0.02508155 1.075757e-04  0.12180542   0.9030531247              0.856
[11,] 0.0257598513 0.02324279 4.929261e-05  0.35851128   0.7199607260              0.736
```

實習教材資源

REQUIRED

【10】熱區分析：LISA、G_i^*與多重檢定校正：FDR

- 授課投影片
- 助教投影片
- LAB10
- 助教課影片
- 空間自相關詳細計算
- 多重檢定校正

空間自相關的詳細計算

REQUIRED

https://www.youtube.com/watch?v=gOuFIxk8oFI&t=876s

>> 套件函數

>> 手動計算

Moran's I

$$I = \frac{n}{W}\frac{\sum_i\sum_j w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\sum_i(x_i-\bar{x})^2}$$

$$\xrightarrow{\tilde{x}_i = x_i - \bar{x}} \frac{n}{W}\frac{\sum_i\sum_j w_{ij}\,\tilde{x}_i\,\tilde{x}_j}{\sum_i \tilde{x}_i^{\,2}}$$

- $W = \sum_i\sum_j w_{ij}$
- $\sum_i (x_i-\bar{x})^2 = n\,\sigma_x^2 = (n-1)\,s_x^2 = (n-1)\,s_x^2$

```
> TP.nb=poly2nb(TP)
> TP.nb.w=nb2listw(TP.nb)
> M=moran.test(x,TP.nb.w)
> M$estimate[1]
Moran I statistic
      -0.01261841
> TP.nb.M=nb2mat(TP.nb)
> xx=x-mean(x)
> sum(TP.nb.M*(xx%*%t(xx)))/sum(xx^2)
[1] -0.01261841
> sum(TP.nb.M*(xx%*%t(xx)))/(var(xx)*11)
[1] -0.01261841
```

$$I = \frac{n}{W}\frac{\sum_i\sum_j w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\sum_i(x_i-\bar{x})^2}$$

$$= \frac{\not{n}}{W}\frac{\sum_i\sum_j w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\not{n}\sigma^2}$$

$$= \frac{1}{W}\sum_i\sum_j w_{ij}\frac{(x_i-\bar{x})}{\sigma}\frac{(x_j-\bar{x})}{\sigma}$$

$$= \frac{1}{W}\sum_i\sum_j w_{ij}z_i z_j$$

$$= \frac{1}{W}\sum_i z_i \sum_j w_{ij}z_j = \frac{1}{W}\sum_i I_i$$

Local Moran's I

$$I_i = z_i \sum_j w_{ij} z_j$$

$$I_i = \frac{x_i-\bar{x}}{s^2}\sum_{j\neq i} w_{ij}(x_j-\bar{x}) = \mathbb{z}_i \sum_j w_{ij}\mathbb{z}_j$$

- $z_i = \frac{x_i-\bar{x}}{\sigma}$
- $\mathbb{z}_i = \frac{x_i-\bar{x}}{s}$

```
> LISA=localmoran(x,TP.nb.w)
> LISA[1];            sum(LISA[,1])/12
[1] 0.005094452       [1] -0.01261841
> z=(x-mean(x))/(sd(x)*sqrt(11/12))
> z[1]*sum(TP.nb.M[1,]*z)
[1] 0.005094452
> LISA=localmoran(x,TP.nb.w,mlvar=F)
> LISA[1]
[1] 0.004669914
> z=(x-mean(x))/sd(x)
> z[1]*sum(TP.nb.M[1,]*z)
[1] 0.004669914
```

補充：用矩陣方法一次求得所有I_i

```
> z*(TP.nb.M%*%z)
```

P.S.

$$I_i = \frac{x_i-\bar{x}}{s_i^2}\sum_{j\neq i} w_{ij}(x_j-\bar{x})\,;\; s_i^2 = \frac{\sum_{j\neq i} w_{ij}(x_j-\bar{x})^2}{n-1}$$

```
> lx=xx[1]*sum(TP.nb.M[1,]*xx)
> si2=var(x[-1])*10/11
> lx/si2
[1] 0.004670523
```

矩陣方法：

```
> xx*(TP.nb.M%*%xx)/ sapply(1:12,
      function(i) var(x[-i])*10/11)
```

Getis-Ord General G

$$G = \frac{\sum_i\sum_j w_{ij}x_i x_j}{\sum_i\sum_j x_i x_j}, j \neq i$$

$$\left(當 w_{ii} = 0 \xrightarrow{\text{ignore } j\neq i} G = \frac{\sum_i\sum_j w_{ij}x_i x_j}{\sum_i\sum_j x_i x_j - \sum_i x_i^2}\right)$$

```
> G=globalG.test(x,TP.nb.w)
> G$estimate[1]
Global G statistic
        0.09243927
> G.num=sum(TP.nb.M*(x%*%t(x)))
> G.den=sum(x%*%t(x))-sum(x^2)
> G.num/G.den
[1] 0.09243927
```

R package – spdep

$$I = \frac{n}{\sum_{i=1}^n\sum_{j=1}^n w_{ij}}\frac{\sum_{i=1}^n\sum_{j=1}^n w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\sum_{i=1}^n(x_i-\bar{x})^2}$$

$$I_i = \frac{(x_i-\bar{x})}{\sum_{k=1}^n(x_k-\bar{x})^2/(n-1)}\sum_{j=1}^n w_{ij}(x_j-\bar{x})$$

```
localmoran(mlvar=TRUE)
```

mlvar: values of local Moran's I are reported using the variance of the variable of interest (sum of squared deviances over n), but can be reported as the sample variance, dividing by (n-1) instead

Getis-Ord Gi*

$$G_i^* = \frac{\sum_j w_{ij}x_j}{\sum_j x_j}$$

Getis-Ord Gi

$$G_i = \frac{\sum_j w_{ij}x_j}{\sum_j x_j}, j \neq i$$

```
> Gi.=localG(x,TP.nb.w.in,return_internals=T)
> attr(Gi.,"internals")[,1]
0.0862 0.0885 0.0923 0.0868 0.0845 ……
> TP.nb.M.in%*%x/sum(x)
0.0862 0.0885 0.0923 0.0868 0.0845 ……
```

```
> Gi=localG(x,TP.nb.w,return_internals=T)
> attr(Gi,"internals")[,1]
0.0946 0.0966 0.0969 0.0948 0.0948 ……
> TP.nb.M%*%x/(sum(x)-x)
0.0946 0.0966 0.0969 0.0948 0.0948 ……
```

空間自相關計算

https://youtu.be/gOuFIxk8oFI

14:38 ~ 46:20

熱區分析的空間統計方法：第二部分

- Host-spot analysis (for polygon data)
  - Local Moran’s I index
  - Local G-statistic (Gi*)
- Issues of multiple testing for hot-spot analysis
  - Bonferroni correction
  - False discovery rate (FDR)

Textbook Chapter

TEXT_Local.Stat.pdf

Chapter 8

Local Statistics

CHAPTER OBJECTIVES

In this chapter, we:

- Explain the concepts underlying the emerging array of *local statistics*
- Account for the relatively late arrival of local statistics on the spatial analytic scene
- Review the various approaches that can be used to construct *localities* for the development of local statistics
- Discuss how the popular Getis-Ord family of G statistics are calculated and interpreted
- Outline the local version of Moran's I statistic
- Explain why inference based on local statistics is challenging and describe current approaches to dealing with the difficulties
- Provide an overview of the increasingly popular method *geographically weighted regression*
- Explain how many other spatial analysis methods can be considered as local statistics even if this was not the intent behind their original development

Chap 8: Local Statistics

p.223 - p.226

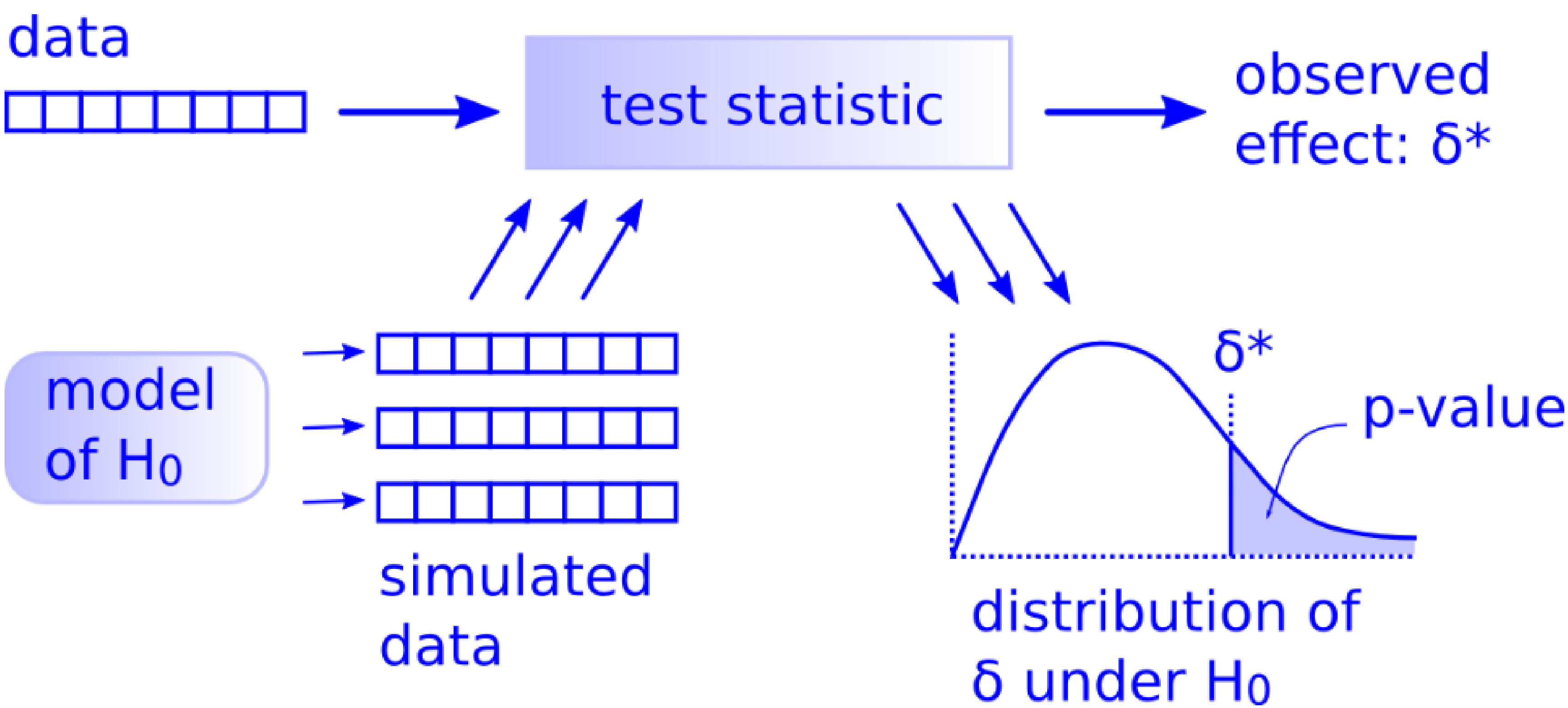
8.1 Introduction: Think geographically, measure locally

8.2 Defining the local

8.3 An example

8.4 Inference with local statistics

熱區的統計顯著性校正
Controlling the False Discovery Rate (FDR)



Recap: Type I Error and Significance Level

- **Type I error** (false positive 偽陽性):

  the incorrect rejection of a true null hypothesis

- **Significance level** (or rate of Type I error):

  the probability of rejecting the null hypothesis

  given that it is true.

Recap: Global vs. Local Measures in Spatial Analysis

- **Global measures** consider all available locations simultaneously, utilizing a single statistic that summarizes the spatial pattern.
- **Local measures** represent the association between each location and its neighbors based on defined distances.

  One statistic is provided for each location, facilitating the identification of clusters, testing of stationarity assumptions, and inference about distances over which spatial association occurs.

Statistical Issues for Local Measures

- Local statistics rely on tests of spatial association for each location in the data, and the issues of multiple comparisons and spatial dependency are the concerns when assessing their **significance**

Multiple Comparisons (or Multiple Testing)

Setting the significance level = 0.05 (rate of Type I Error)

REAL

Spatial Statistical Results

A total of polygons (N) = 1050

TRUE Random = 1000 (null)

TRUE Clusters = 50 (reject null)

A total of polygons (N) = 1050
Accept null hypothesis (n1 = 950):
Random = 1000 * 0.95 = 950

Reject null hypothesis (n2 = 100):
Clusters (Type I error) = 1000 * 0.05 = 50
TRUE Clusters = 50 (true positive)

The local analysis identifies 100 polygons as clusters!
However, **HALF** of them are WRONG !!!

How to make sure the rate of Type I Error = 0.05

$$0.05 = 1 - (1 - \alpha)^{n}$$

$$= 1 - (1 - \alpha)^{100}$$

$$\alpha = 0.000153$$

Adjusting for multiple comparisons (cont'd)

Sidak Correction (1967, 1968, 1971)

The Sidak correction controls for **the overall probability of type I error,** but with critical values appraised at a level $1-(1-\alpha)^{1/n}$. Therefore, a test is considered significant when $p \leqq p_{critical} = 1-(1-\alpha)^{1/n}$

Weakness: usually produce **conservative** results.

Adjusting for multiple comparisons

The Bonferroni method

It evaluates the significance of the test statistics at a critical probability value ($p_{critical}$) set equal to α/n, where α is the overall type I error rate for the data.

All test statistics whose probability values (p) satisfy the condition $p \leqq p_{critical} = \alpha / n = p_{BON}$ are considered significant (null hypothesis is rejected)

Weakness: usually produce **conservative** results.

Rationale for alternative approach

- **Ignoring the issue of multiple testing** would imply spending a large amount of human and financial resources *unnecessarily and inefficiently*.
- **The extremely conservative methods** would result in a *major failure* to curb the spread of the disease (or crime events).

Adjusting for multiple comparisons: The false discovery rate (FDR)

Benjamini and Hochberg (1995)

step-by-step procedure:

Assume that there are *m* hypotheses to be tested

(1) order the test statistics p-values (p_i)

in ascending order ($p_1 \leqq p_2 \leqq . . . \leqq p_m$);

(2) starting from p_m find the first p_i for which $p_i \leqq p_{critical} = (i/m)\,\alpha$;

(3) regard all tests as significant for which $p_i \leqq p_{critical} = (i/m)\,\alpha = p_{FDR}$.

Comparisons of different corrections

Year	Unadjusted	Correcting for multiplicity*			Correcting for multiplicity and spatial dependence†		Recovery ratio‡
		Bonferroni	Sidak	FDR	Bonferroni	Sidak	
1987 (N = 740)							
$G_i^*(d)$							
Accept null	387	665	664	436	650	649	
Reject null	353	75	76	304	90	91	
Cluster high rates	142	20	21	116	28	28	0.824
Cluster low rates	211	55	55	188	62	63	
$p_{critical}$	0.025	0.0000338	0.0000342	0.0100272	0.0000616	0.0000624	
$z_{critical}$	± 1.95996	± 3.98469	± 3.98169	± 2.32533	± 3.83958	± 3.83648	
$G_i(d)$							
Accept null	392	675	675	437	659	659	
Reject null	348	65	65	303	81	81	
Cluster high rates	136	16	16	116	21	21	0.841
Cluster low rates	212	49	49	187	60	60	
$p_{critical}$	0.025	0.0000338	0.0000342	0.0101780	0.0000616	0.0000624	
$z_{critical}$	± 1.95996	± 3.98469	± 3.98169	± 2.31972	± 3.83958	± 3.83648	
Moran's I_i							
Accept null	439	740	740	479	740	740	
Reject null	301	0	0	261	0	0	
Cluster high rates	100	0	0	83	0	0	0.867
Cluster low rates	201	0	0	178	0	0	
$p_{critical}$	0.05	0.0000676	0.0000693	0.0220000	0.0001232	0.0001264	
$z_{critical}$	1.64485	3.81691	3.81061	2.01409	3.66588	3.65936	

Comparisons of different corrections (cont'd)

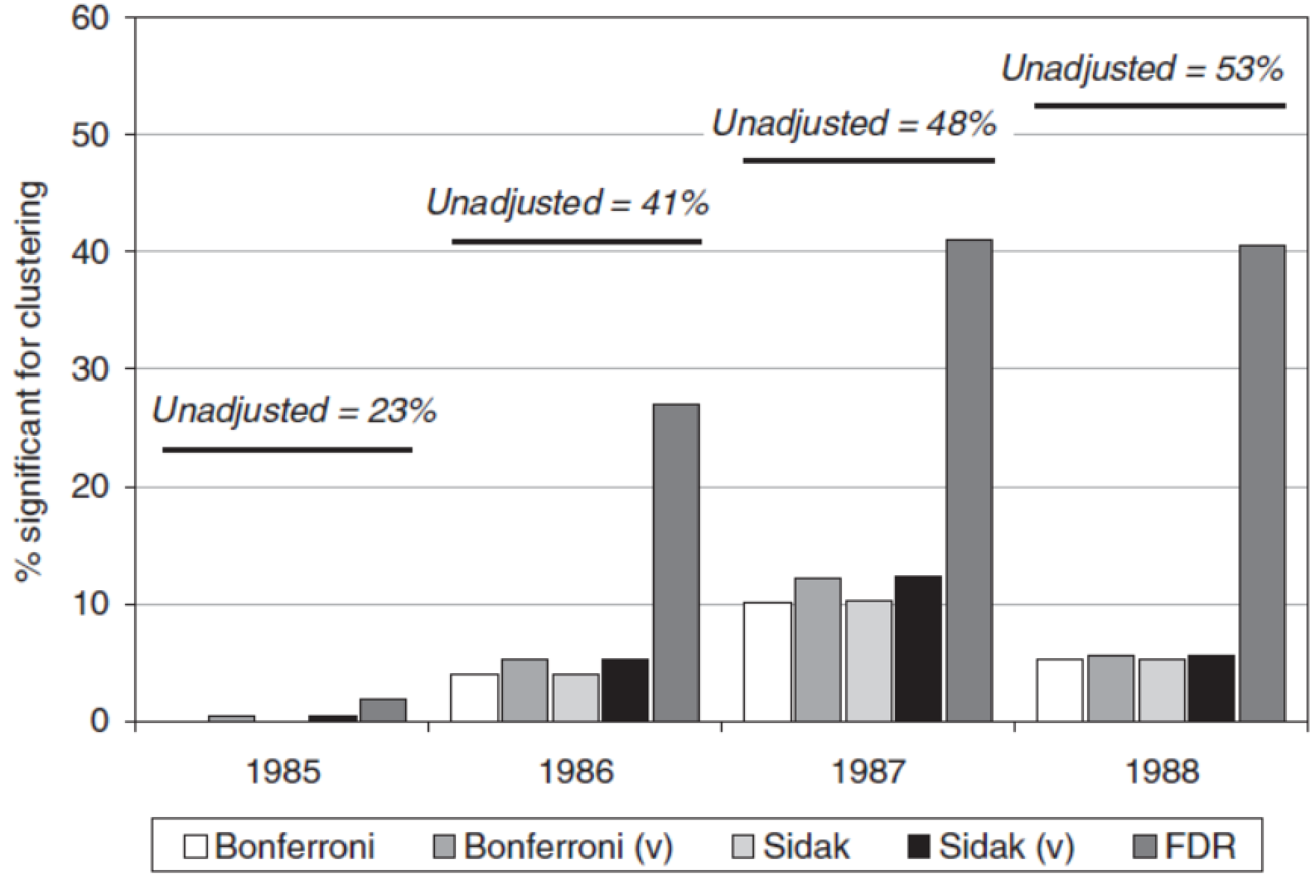
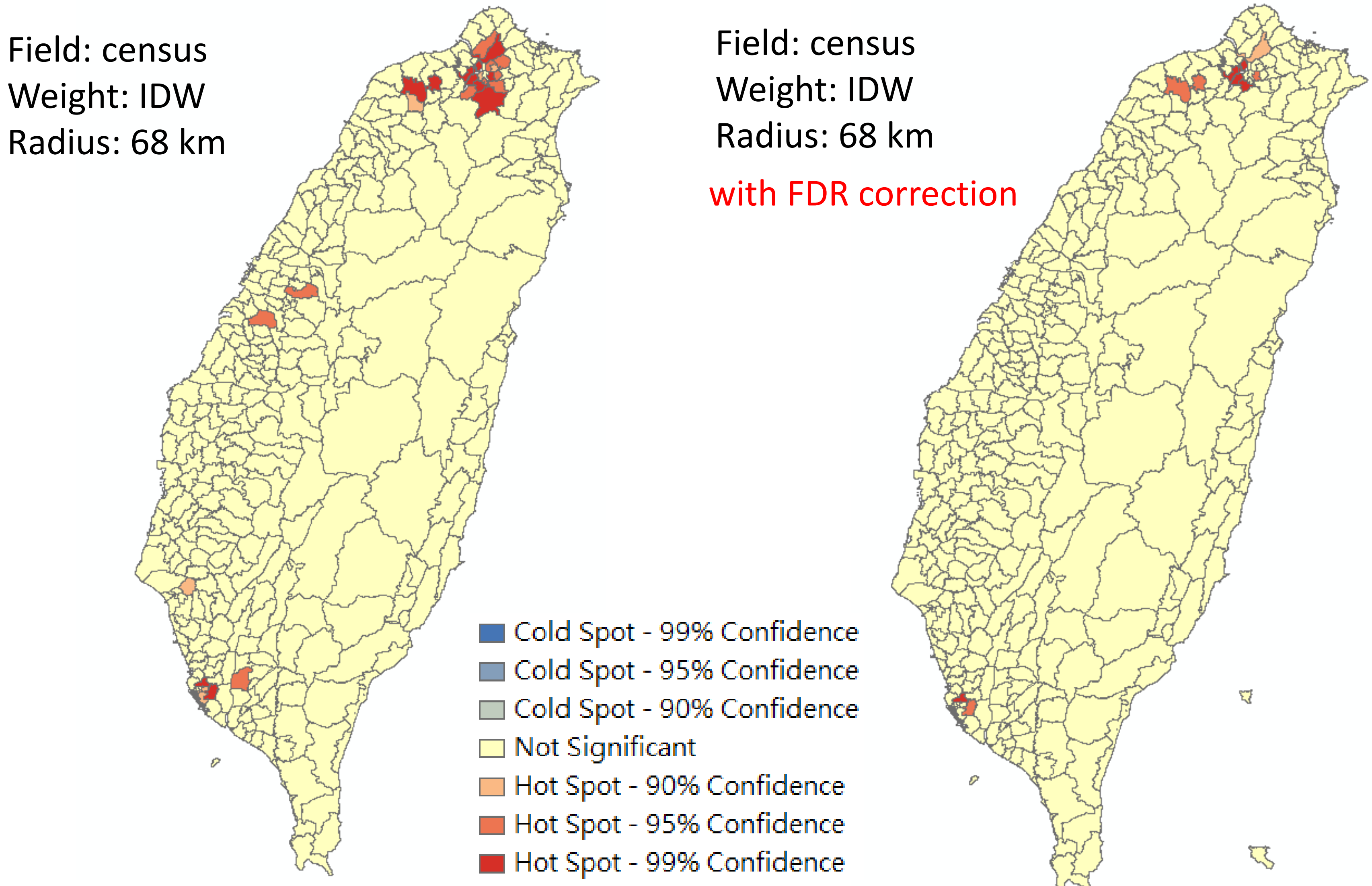


Figure 2. Percentage of plots that tested significant for clustering, according to the $G_i^*(d)$ statistic and different control procedures—Machadinho (1985/95).

Result: the hotspots of population using Gi*(d)



Hotspots with/without FDR correction

Gi_IDW

OID	OBJECTID	SOURC	CENSUS	GiZScore	GiPValue	Gi_Bin
55	56	55	516468	6.020918	0	3
46	47	46	401619	4.501931	0.000007	3
43	44	43	384051	4.269309	0.00002	3
63	64	63	376584	4.170142	0.00003	3
228	229	228	357536	3.912712	0.000091	3
220	221	220	338361	3.661243	0.000251	3
218	219	218	329913	3.548686	0.000387	3
262	263	262	322678	3.451571	0.000557	3
31	32	31	315818	3.367603	0.000758	3
29	30	29	292096	3.050694	0.002283	3
62	63	62	272500	2.790864	0.005257	3
34	35	34	254521	2.553956	0.010651	2
35	36	35	253920	2.546308	0.010887	2
36	37	36	247904	2.465707	0.013674	2
38	39	38	237530	2.331119	0.019747	2
45	46	45	231938	2.255883	0.024078	2
352	353	352	231129	2.239411	0.025129	2
49	50	49	229383	2.225159	0.026071	2
1	2	1	221815	2.116672	0.034288	2
32	33	32	216043	2.047546	0.040604	2
179	180	179	215245	2.028891	0.042469	2
37	38	37	205031	1.901785	0.057199	1
40	41	40	204024	1.890186	0.058733	1
232	233	232	203001	1.868696	0.061665	1
233	234	233	199535	1.823772	0.068186	1
103	104	103	198372	1.805552	0.070988	1
219	220	219	194521	1.757798	0.078782	1
93	94	93	185752	1.638691	0.101278	0
294	295	294	177796	1.534813	0.12483	0

< 0.01

< 0.05

< 0.1

with FDR correction

Gi_IDW_FDR

OID	OBJECTID	SOURCE_ID	CENSUS	GiZScore	GiPValue	Gi_Bin
55	56	55	516468	6.020918	0	3
46	47	46	401619	4.501931	0.000007	3
43	44	43	384051	4.269309	0.00002	3
63	64	63	376584	4.170142	0.00003	3
228	229	228	357536	3.912712	0.000091	3
220	221	220	338361	3.661243	0.000251	2
218	219	218	329913	3.548686	0.000387	2
262	263	262	322678	3.451571	0.000557	2
31	32	31	315818	3.367603	0.000758	2
29	30	29	292096	3.050694	0.002283	1
62	63	62	272500	2.790864	0.005257	0
34	35	34	254521	2.553956	0.010651	0
35	36	35	253920	2.546308	0.010887	0
36	37	36	247904	2.465707	0.013674	0
38	39	38	237530	2.331119	0.019747	0
45	46	45	231938	2.255883	0.024078	0
352	353	352	231129	2.239411	0.025129	0
49	50	49	229383	2.225159	0.026071	0
1	2	1	221815	2.116672	0.034288	0
32	33	32	216043	2.047546	0.040604	0
179	180	179	215245	2.028891	0.042469	0
37	38	37	205031	1.901785	0.057199	0
40	41	40	204024	1.890186	0.058733	0
232	233	232	203001	1.868696	0.061665	0
233	234	233	199535	1.823772	0.068186	0
103	104	103	198372	1.805552	0.070988	0
219	220	219	194521	1.757798	0.078782	0
93	94	93	185752	1.638691	0.101278	0
294	295	294	177796	1.534813	0.12483	0

FDR correction

$$p_i \le p_{critical} = (i/m)\alpha = p_{FDR}.$$

m = 361

	A	B	C	F	G	H	I	J	K	L	M	N	O
1	OBJECTII	SOURCE_	**CENSUS**	**GiZScore**	**GiPValue**	**Gi_Bin**	Rank	**P_FDR_0.1**	**P_FDR_0.05**	**P_FDR_0.01**	**Sig_0.1**	**Sig_0.05**	**Sig_0.01**
2	56	55	516468	6.021	**0.000000002**	3	1	0.000277008	0.000138504	2.77008E-05	Sig	Sig	**Sig**
3	47	46	401619	4.502	**0.000006734**	3	2	0.000554017	0.000277008	5.54017E-05	Sig	Sig	**Sig**
4	44	43	384051	4.269	**0.000019608**	3	3	0.000831025	0.000415512	8.31025E-05	Sig	Sig	**Sig**
5	64	63	376584	4.170	**0.000030441**	3	4	0.001108033	0.000554017	0.000110803	Sig	Sig	**Sig**
6	229	228	357536	3.913	**0.000091265**	3	5	0.001385042	0.000692521	0.000138504	Sig	Sig	**Sig**
7	221	220	338361	3.661	**0.000250994**	3	6	0.00166205	0.000831025	0.000166205	Sig	**Sig**	FALSE
8	219	218	329913	3.549	**0.000387159**	3	7	0.001939058	0.000969529	0.000193906	Sig	**Sig**	FALSE
9	263	262	322678	3.452	**0.000557333**	3	8	0.002216066	0.001108033	0.000221607	Sig	**Sig**	FALSE
10	32	31	315818	3.368	**0.000758246**	3	9	0.002493075	0.001246537	0.000249307	Sig	**Sig**	FALSE
11	30	29	292096	3.051	**0.002283134**	3	10	0.002770083	0.001385042	0.000277008	**Sig**	FALSE	FALSE
12	63	62	272500	2.791	**0.005256756**	3	11	0.003047091	0.001523546	0.000304709	FALSE	FALSE	FALSE
13	35	34	254521	2.554	**0.010650667**	2	12	0.0033241	0.00166205	0.00033241	FALSE	FALSE	FALSE
14	36	35	253920	2.546	**0.010886894**	2	13	0.003601108	0.001800554	0.000360111	FALSE	FALSE	FALSE
15	37	36	247904	2.466	**0.013674312**	2	14	0.003878116	0.001939058	0.000387812	FALSE	FALSE	FALSE
16	39	38	237530	2.331	**0.019747089**	2	15	0.004155125	0.002077562	0.000415512	FALSE	FALSE	FALSE
17	46	45	231938	2.256	**0.024077970**	2	16	0.004432133	0.002216066	0.000443213	FALSE	FALSE	FALSE
18	353	352	231129	2.239	**0.025129188**	2	17	0.004709141	0.002354571	0.000470914	FALSE	FALSE	FALSE
19	50	49	229383	2.225	**0.026070581**	2	18	0.00498615	0.002493075	0.000498615	FALSE	FALSE	FALSE
20	2	1	221815	2.117	**0.034287671**	2	19	0.005263158	0.002631579	0.000526316	FALSE	FALSE	FALSE
21	33	32	216043	2.048	**0.040604468**	2	20	0.005540166	0.002770083	0.000554017	FALSE	FALSE	FALSE
22	180	179	215245	2.029	**0.042469389**	2	21	0.005817175	0.002908587	0.000581717	FALSE	FALSE	FALSE
23	38	37	205031	1.902	**0.057199259**	1	22	0.006094183	0.003047091	0.000609418	FALSE	FALSE	FALSE
24	41	40	204024	1.890	**0.058733043**	1	23	0.006371191	0.003185596	0.000637119	FALSE	FALSE	FALSE
25	233	232	203001	1.869	**0.061665141**	1	24	0.006648199	0.0033241	0.00066482	FALSE	FALSE	FALSE
26	234	233	199535	1.824	**0.068186495**	1	25	0.006925208	0.003462604	0.000692521	FALSE	FALSE	FALSE
27	104	103	198372	1.806	**0.070988331**	1	26	0.007202216	0.003601108	0.000720222	FALSE	FALSE	FALSE
28	220	219	194521	1.758	**0.078781940**	1	27	0.007479224	0.003739612	0.000747922	FALSE	FALSE	FALSE
29	94	93	185752	1.639	0.101277593	0	28	0.007756233	0.003878116	1.07427E-06	FALSE	FALSE	FALSE
30	295	294	177796	1.535	0.124829823	0	29	0.008033241	0.00401662	1.11264E-06	FALSE	FALSE	FALSE

Using p.adjust() function

p.adjust

Adjust P-Values For Multiple Comparisons

Given a set of p-values, returns p-values adjusted using one of several methods.

Keywords htest

Usage

```
p.adjust(p, method = p.adjust.methods, n = length(p))

p.adjust.methods
# c("holm", "hochberg", "hommel", "bonferroni", "BH", "BY",
#   "fdr", "none")
```

Arguments

p numeric vector of p-values (possibly with `NA` s). Any other R object is coerced by `as.numeric` .

method correction method. Can be abbreviated.

Mapping p-values (localmoran)

```
> LISA.Popn <- localmoran(Density, TWN_nb_w, zero.policy=T)
> LISA.Popn
              Ii    E.Ii     Var.Ii          Z.Ii   Pr(z > 0)
221  0.668654386 -0.025 0.17429750  1.66148935 4.830760e-02
222  0.552267439 -0.025 0.22386865  1.22005786 1.112215e-01
223  1.122969659 -0.025 0.17429750  2.74969696 2.982520e-03
224  0.520540764 -0.025 0.14125006  1.45155294 7.331298e-02
225  1.283663684 -0.025 0.14125006  3.48203973 2.488049e-04
226  1.275561091 -0.025 0.17429750  3.11519460 9.191180e-04
227  2.310784759 -0.025 0.14125006  6.21496221 2.566850e-10
```

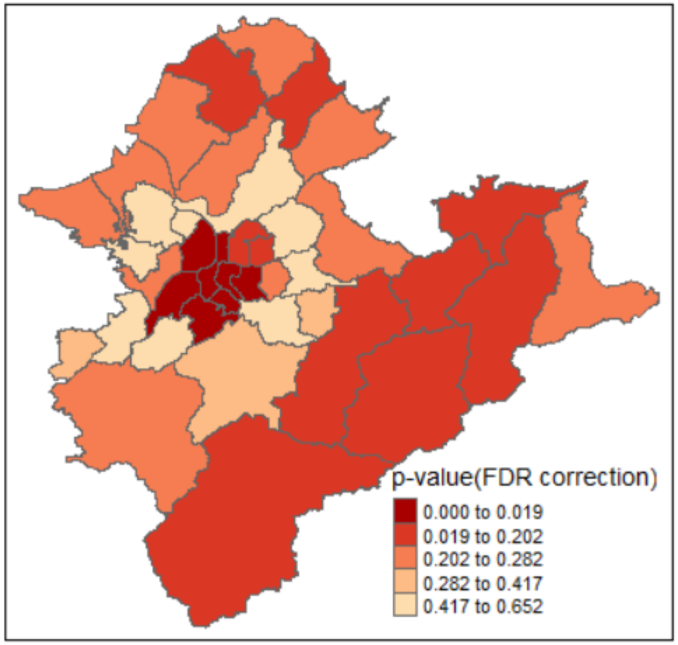
Mapping p-values (FDR correction)

```
LISA.Popn <- localmoran(Density, TWN_nb_w, zero.policy=T)

NorthTW_sf$pvalue.adj <- p.adjust(LISA.Popn[,5], method="fdr")
```

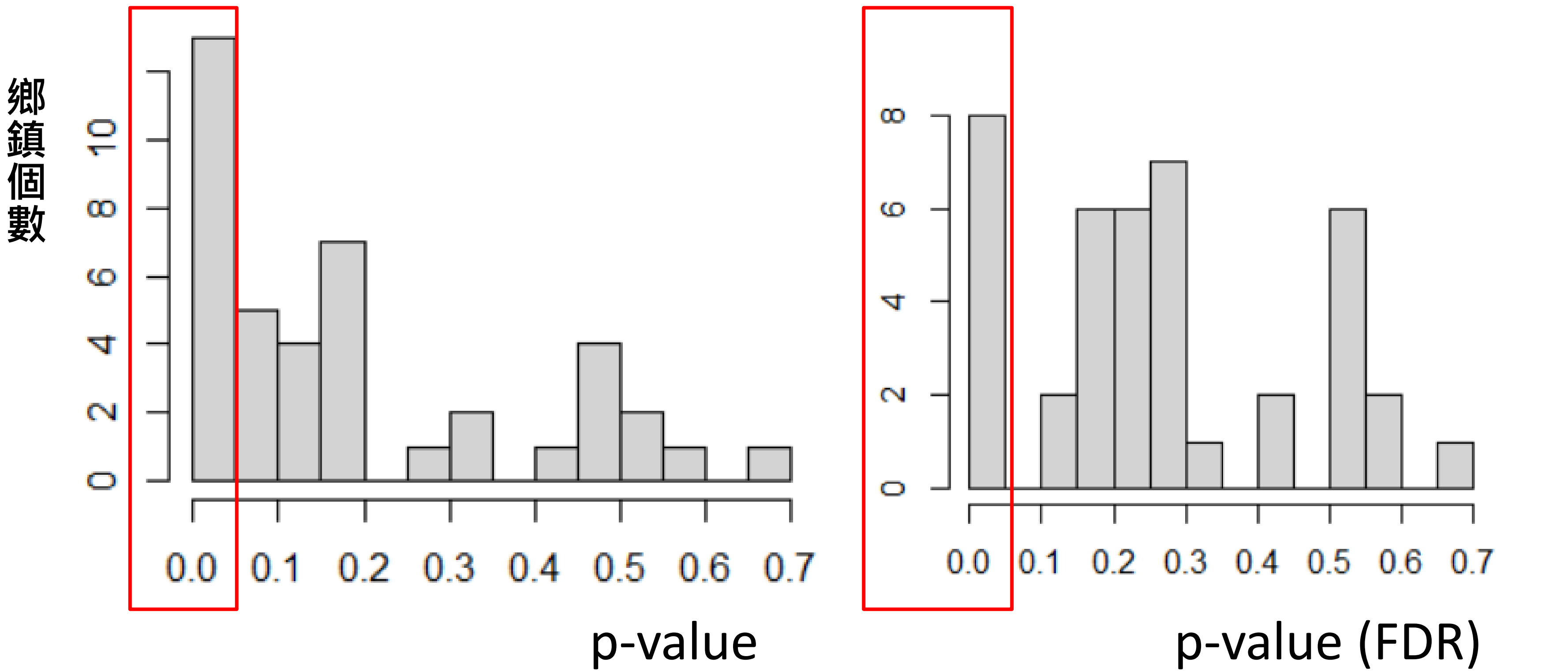
pvalue = (i/41) x pvalue.adj

→ pvalue * (41/i) = pvalue.adj

```
> min(NorthTW_sf$pvalue.adj)
[1] 1.052409e-08
> min(NorthTW_sf$pvalue) * 41
[1] 1.052409e-08

> max(NorthTW_sf$pvalue.adj)
[1] 0.6524963
> max(NorthTW_sf$pvalue)
[1] 0.6524963
```



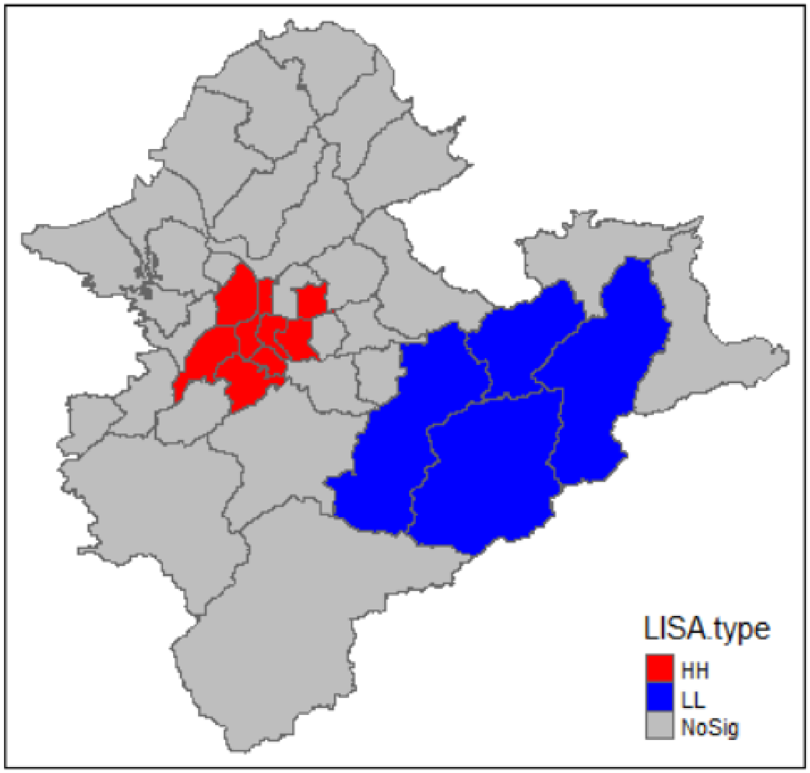
Comparisons of p-values before/after correction



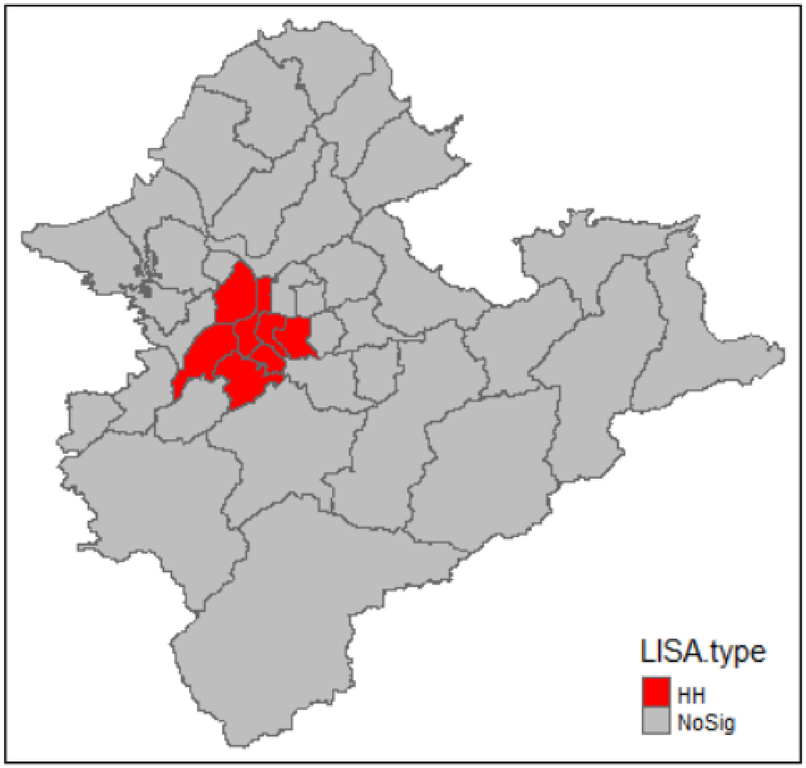
Mapping LISA Maps

alpha = 0.05

LISA map



LISA map (FDR correction)



Mapping Significant Hot-spots (Gi*)

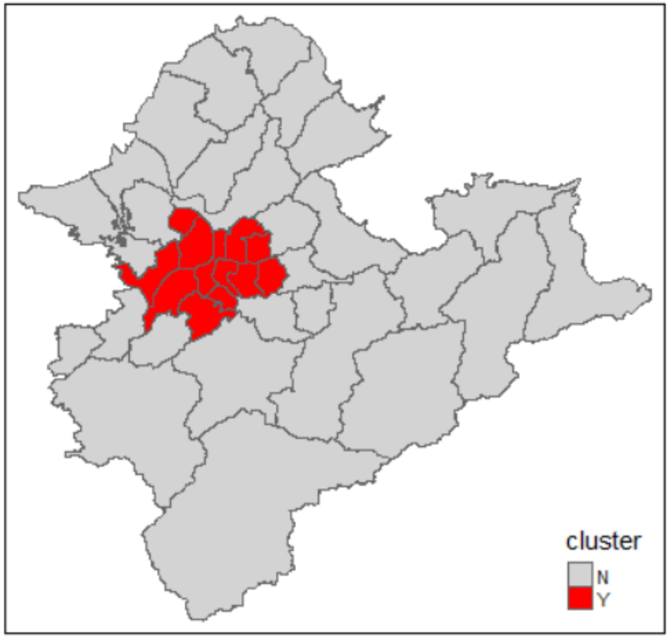
Bonferroni correction

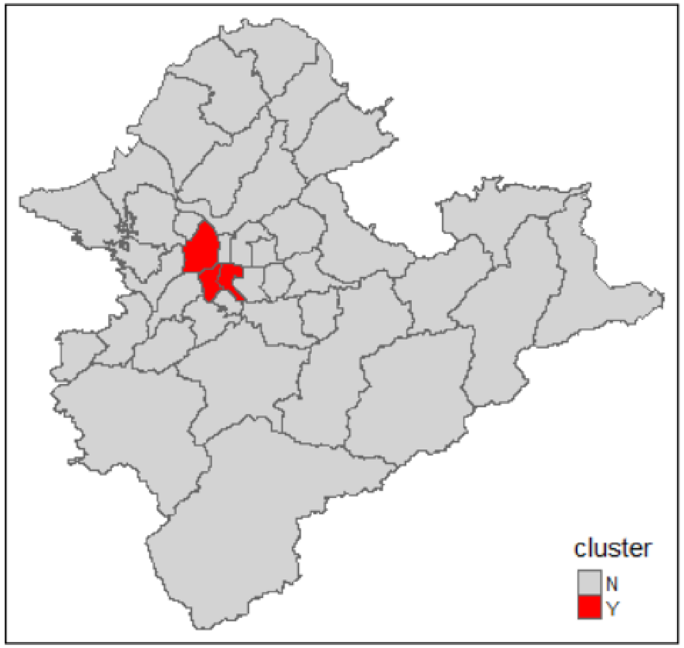
alpha = 0.05

alpha = 0.05/41

$Z^* = 1.65$

$Z^* = 3.03$





p.adjustSP

From spdep v0.8-1
by Roger Bivand

99.99th
Percentile

Adjust Local Association Measures' P-Values

Make an adjustment to local association measures' p-values based on the number of neighbours (+1) of each region, rather than the total number of regions.

Keywords spatial

Usage

```
p.adjustSP(p, nb, method = "none")
```

Arguments

p vector of p-values

nb a list of neighbours of class `nb`

method correction method as defined in `p.adjust`: "The adjustment methods include the Bonferroni correction ('"bonferroni"') in which the p-values are multiplied by the number of comparisons. Four less conservative corrections are also included by Holm (1979) ('"holm"'), Hochberg (1988) ('"hochberg"'), Hommel (1988) ('"hommel"') and Benjamini & Hochberg (1995) ('"fdr"'), respectively. A pass-through option ('"none"') is also included."

Comparisons: p.adjust vs. p.adjustSP

```
NorthTW_sf$pvalue.adj2.1 <-
p.adjust (LISA.Popn[,5], method="fdr")

NorthTW_sf$pvalue.adj2.2 <-
p.adjustSP(LISA.Popn[,5], TWN_nb, method="fdr")
```

Comparisons: p.adjust vs. p.adjustSP

	未校正的p-value		p.adjust	p.adjustSP
	pvalue	pvalue.adj	pvalue.adj2.1	pvalue.adj2.2
221	0.1720592912	0.32065595	0.32065595	1.000000000
222	0.2134934671	0.33666277	0.33666277	1.000000000
223	0.0576411094	0.19798039	0.19798039	0.345846656
224	0.0312678035	0.18313999	0.18313999	0.218874624
225	0.0005928042	0.01215249	0.01215249	0.004149629
226	0.0460037835	0.19212781	0.19212781	0.276022701

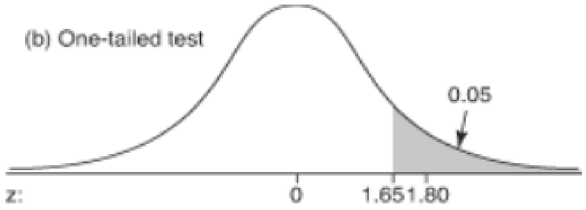
Using localg() function for adjustment

spdep (version 1.1-7)

localG: G and Gstar local spatial statistics

Description

The local spatial statistic G is calculated for each zone based on the spatial weights object used. The value returned is a Z-value, and may be used as a diagnostic tool. High positive values indicate the posibility of a local cluster of high values of the variable being analysed, very low relative values a similar cluster of low values. For inference, a Bonferroni-type test is suggested in the references, where tables of critical values may be found (see also details below).



The critical values of the statistic under assumptions given in the references for the 95th percentile are for n=1: 1.645, n=50: 3.083, n=100: 3.289, n=1000: 3.886.

The Bonferroni correction method

The critical values of the statistic under assumptions given in the references for the 95th percentile are for n=1: 1.645, n=50: 3.083, n=100: 3.289, n=1000: 3.886.

```
> qnorm(1-0.05, 0, 1)
[1] 1.644854
> qnorm(1-0.05/50, 0, 1)
[1] 3.090232
> qnorm(1-0.05/100, 0, 1)
[1] 3.290527
> qnorm(1-0.05/1000, 0, 1)
[1] 3.890592
```